PLASTIC ANALYSIS AND DESIGN OF AXISYMMETRIC FOOTING SLABS BY YIELD EQUALITY METHOD

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By
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TO

MY LOVING PARENTS AND RELATIVES,
FOR THEIR COOPERATION, ENCOURAGEMENT
AND UNDERSTANDING .

CERTIFICATE

This is to certify that the thesis entitled ' PLASTIC ANALYSIS AND DESIGN OF AXISYMMETRIC FOOTING SLABS BY YIELD EQUALITY METHOD ' by JOSHI MADHAV HARI is a record of work carried out under my supervision and that it has not been submitted elsewhere for a degree.

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A B S T R A C T

The relevance of plastic analysis in the Structural Engineering world has been well established in as much as it reflects the true state of affairs at the ultimate or the collapse stage and gives a clear picture of the margin of safety under working conditions. Solutions obtained by lower bound methods guarantee safe solutions and yield exact solutions also if the kinematic conditions are satisfied. ADIDAM'S "YIELD EQUALITY METHOD" , employing physical reasoning for its application yields satisfactory solutions, which have been corroborated by experimental results. This method is most suitable for axisymmetric slabs.

Column footings can be visualised as slabs subjected to loading from below by the subgrade reactions. Tapered and Stepped footings afford easy means of visualisation of the various yield zones and as such can be conveniently analysed by Yield Equality Method.

This thesis presents the analysis of no. of typical cases. Included in them are slabs with polar net

reinforcement and also isotropic mesh reinforcement. The subgrade reaction also assumes different shapes depending upon the nature of the foundation soils. Purely granular and cohesive soils are considered. The more common uniform reaction adopted in practical cases is also seperately considered. In all 9 different cases are analysed and charts are drawn connecting the moment capacity with radius of the footing expressed in non-dimensionalised form. They can be used as design charts for particular design situations. Some design problems using these charts are worked out.

The analysis can be extended for annular slab footings. Optimisation techniques can also be employed to arrive at optimal taper angles and location of steps using minimum cost criterion.

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M.H. JOSHI

CONTENTS

CHAPTER 1

INTRODUCTION TO PLASTIC ANALYSIS

1.1 GENERAL

It is the responsibility of the Engineers to design structures to be safe under all working conditions. In this endeavour ~~We~~ one should have sufficient margin of safety for unexpected overloads consistant with economy. This has been hitherto done by the application of elastic theory. But in recent times application of plastic methods has also been employed to supplement the elastic theory. Plastic theory has the advantage of reflecting the real state of affairs at the collapse stage. This approach is purely behavioural in character and allows the true margin of safety to be assessed. The elastic approach however is necessary to check up deflections under working loads.

1.2 PLASTIC ANALYSIS - BASIC CONCEPTS

Plastic analysis enables one to know the behaviour of the structure loaded upto the plastic range. The generalised stress-strain relationship of any structure is, if at all, linear only in the initial stages of loading; but at the later stages, especially at ultimate or collapse stage, it is very complicated. But idealised stress-strain

relationships have been assumed in the plastic analysis.

No real material is wholly Hookean (perfectly elastic) or wholly a Saint-Venant substance (rigid perfectly plastic). But they are in general elasto-plastic in character and may be idealised as elastic-perfectly plastic. In respect of steel the stress-strain relationship beyond the workhardening range is neglected and perfect plasticity is assumed. As regards concrete the complicated stress-strain relationship is replaced by an equivalent rectangular stress block. As the deflections during the plastic stage are quite large the initial elastic deflections are ignored in the plastic analysis.

1.2.1 YIELD CONDITION AND FLOW RULE

In the elastic range stress and strain are related by stress-strain relations viz

$$E\,\epsilon^{(e)} = \sigma \quad , \quad G\,\gamma^{(e)} = \tau \qquad (1.1)$$

In the plastic range these relations must be supplemented by yield condition and flow rule. The yield condition expresses all possible combinations of the generalised stresses which produce plastic flow and is also compatible with the assumption that no plastic flow takes place under a hydrostatic system of stresses. The flow rule

gives for any such state of stress the ratios between the increments of plastic strain. The yield condition is expressed as a function that relates the generalised stresses at yield and is given by

$\phi(Q) = 0$ where Q denotes generalised stress such that

$\phi(Q) < 0$ for no yield.

Yield occurs only when $\phi(Q) = 0$ is satisfied and the combinations of stresses corresponding to $\phi(Q) > 0$ are impossible.

The point, curve or surface corresponding to the above equation is known as the yield point, the yield curve or the yield surface respectively. The yield surface can be shown to be always convex.

The flow rule is assumed in such a way that the strain vector q is given as

$$q_i = \lambda \frac{d\phi}{dQ_i} \qquad i = 1,2,\ldots n \qquad (1.2)$$

where λ is an arbitrary factor of proportionality. Above equation implies that the plastic-strain-increment vector is normal to the yield surface at a smooth point and lies between adjacent normals at a corner.

1.2.2 YIELD CRITERIA

Any number of yield criteria are permissible within

the framework of ideal or perfect plasticity. The yield criteria that are described in this section are the Tresca yield criteria, the Von Mises yield criteria and the square yield criteria of Johansen.

TRESCA YIELD CRITERION

It is also known as maximum shearing stress ... n criterion. In terms of principal stresses σ_1, σ_2, for the plane stress case, it is given by

$$\text{Max.}(|\sigma_1|, |\sigma_2|, |\sigma_1 - \sigma_2|) = \sigma_y \tag{1.3}$$

This is represented by a Hexagon as shown in the Fig. 1.1(a)

MISE'S YIELD CRITERION

This was introduced by Mises on mathematical grounds and interpreted later by Hencky & Huber that plastic flow occurs when the shear strain energy stored reaches a definite value. This is represented by anellipse passing through the corners of the Hexagon as shown in Fig. 1.1(b). Equation of this ellipse is

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \sigma_y^2 \tag{1.4}$$

SQUARE YIELD CRITERION

This yield criterion was intuitively adopted by Johansen (1943,1) in the plastic analysis of reinforced

concrete plates. For an isotropic slab in which there is equal amount of reinforcement at top and bottom, in terms of principal stresses the square yield criterion is given by

$$|\sigma_i| = \sigma_y \qquad i = 1,2 \tag{1.5}$$

The yield curve and flow rule are shown in Fig. 1.1(c)

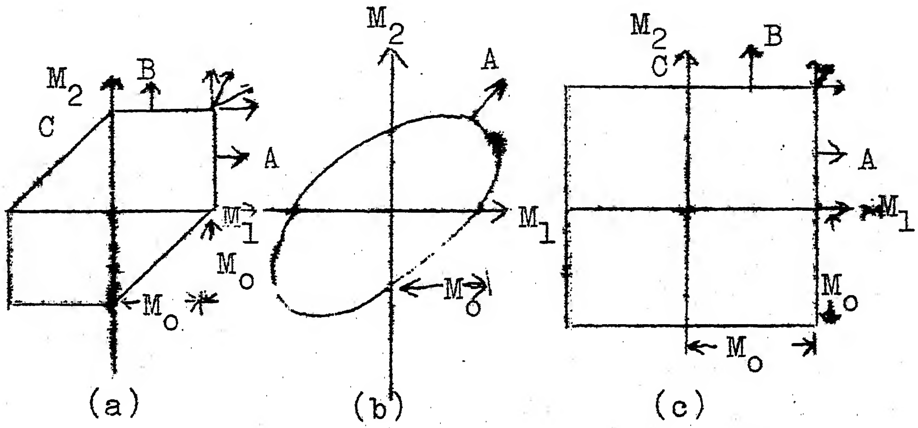


Fig. 1.1 YIELD CRITERIA

1.2.3 FUNDAMENTAL THEOREMS OF LIMIT ANALYSIS

A more general treatment of structures requires a statement of the limit theorems of Drucker, Greenberg and Prager (1952,2).

LOWER BOUND THEOREM (PRAGER (1959,1))

If an equilibrium distribution of stress can be found which balances the applied load and is every where below yield or at yield, the structure will not collapse or will just be at the point of collapse.

UPPER BOUND THEOREM (PRAGER (1959,1))

The structure will collapse if there is any

compatible pattern of plastic deformation for which the rate at which the external forces do work exceeds the rate of internal dissipation.

The maximum lower bound and minimum upper bound on load capacity are nothing but the limit load itself.

CHAPTER 2

2.1 SELECTIVE LITERATURE SURVEY

INGERSLEV (1923,1) assuming constant bending moments along yield lines gave a method to calculate the strength of a rectangular slab.

GVOZDEV (1936,1) has given probably the first proof of the theorems of limit analysis for proportional loading. THe introduced many concepts now familiar in plasticity theory, notably generalized forces and displacements, the principle of maximum plastic work and the equivalent normality condition for strain rates at yield. This concept was employed in 1952 by Prager when establishing a general theory of limit design, and Hodge has given it a central place in his text on the plastic analysis of structures. He mainly dealt with the reinforced concrete slabs.

JOHANSEN (1943,1) besides the basic introductory theory provided solutions for a large variety of practical problems in the field of plastic analysis of reinforced concrete slabs. The introduction of yield-line theory - the two alternatives: the 'Work Method' and the 'Energy Method', stands out as the most significant contribution to the theory of perfectly plastic solids. He introduced the concept of Nodal forces acting at the junction of yield lines, which was overlooked

by Ingerslev. Although Johansen did not establish rigorously the yield condition for orthotropically reinforced slabs, only the 'square yield' criterion for the isotropic slabs, he proposed a method intuitively to calculate the collapse load of the slabs. This nodal force theory is also known as the 'equilibrium method' and gives only an upper bound on the collapse load since equilibrium is not necessarily satisfied throughout the domain of the slab, but only for boundries of the plate segments separated by yield lines. A complete plastic limit Analysis (Prager and Hodge (1951)) gives identical lower and upper bounds to the correct collapse load and requires the equilibrium to be satisfied at all points of the domain.

ROZVANY et al (1969,1) introduced a method of limit analysis for axisymmetric slabs and shells which has advantages over the usual upper bound (yield line) methods. Instead of finding the worst failure mechanism, it is only necessary to find a stress field that satisfies the equilibrium and certain yield equalities.

A comprehensive set of solutions is given for axisymmetric slabs. It is shown that some solutions in the literature derived on the basis of yield line methods are erroneous.

ROZVANY AND MELCHERS (1970,1) presented a new approach to the direct design of axisymmetric slabs. They derived a set of general rules for the form of optimal solutions and gave a comprehensive set of least reinforcement solution. Examples were given to illustrate the method showing that this approach results in a considerable saving in material.

ADIDAM (1972,1) presented an exhaustive literature survey and discussed a new method of plastic limit analysis termed "the yield equality method" to analyse a variety of axisymmetric reinforced concrete slabs with different layouts of reinforcement. Design charts were also presented. ADIDAM used this method to design and study the post-yield behaviour of footing slabs subjected to various pressure distributions. He conducted tests on fifteen circular slabs with different boundary and loading conditions to prove the validity of the foregoing method.

CHAPTER 3

ANALYSIS OF AXISYMMETRIC FOOTING SLABS

3.1 YIELD EQUALITY METHOD

The collapse loads of axisymmetric slabs having axisymmetric loading, reinforcement and boundary conditions are determined in this chapter. The yield line method pioneered by Johansen (1943,1) has been used extensively for the simpler axisymmetric problems. In the case of non-axisymmetric slabs the correct yield line pattern is known for a few non-trivial cases only. Most of the solutions known are for cases in which total collapse occurs. In such cases, at every point of the slab atleast one yield inequality must be satisfied as a yield equality, since only then does the flow rule of plasticity permit the circumferential curvature rate to be non-zero.

A theorem proposed and proved by Rozvany(1970,2) provides a unified approach for the determination of collapse loads for all types of collapse mechanisms, whether partial or total. This theorem is based on the lower bound theorem of limit analysis. In an axisymmetric plate with a rectangular yield criterion, let the circumferential plastic moment capacity (yield moment) be $\cancel{\overline{M}_\theta}$

$\overline{M}_\theta$ for positive bending and $\alpha \overline{M}_\theta$ for negative bending and the radial yield moments $\overline{M}_r$ & $\beta \overline{M}_r$. In the theorem that follows, P(r) is the axially symmetric load.

$\bar{M}_\theta$, $\bar{M}_r$, α and β are specific functions of radius r which are piecewise continuous and bounded and the collapse load is $\lambda_o P(r)$ where λ_o is the highest statically admissible multiplier.

THEOREM

"The correct load capacity $\lambda_o P(r)$ of an axisymmetric slab is always associated with at least one piecewise continuous safe statically admissible moment field such that $M_\theta = \bar{M}_\theta$ or $M_\theta = -\alpha \bar{M}_\theta$ throughout the slab."

The moment field is said to be statically admissible if it satisfies the equilibrium equation.

$$(r.M_r)'' - M_\theta' = - P(r).r \qquad (3.1)$$

where prime denotes the differentiation with respect to radius r, and it is called safe if it satisfies the yield inequalities.

$$-\alpha \bar{M}_\theta < M_\theta < \bar{M}_\theta$$

$$-\beta \bar{M}_r < M_r < \bar{M}_r$$

This theorem by itself does not constitute a sufficient condition for the correct load capacity unless a corresponding kinematically admissible mechanism exists. The existence of such a mechanism can be verified by inspection of the moment diagram. In the case of partial collapse,

in rigid regions of the slab an infinite number of safe statically admissible moment fields can be determined with the aid of this theorem. Rozvany gave a proof for the case $\alpha = \beta = 1$, and Rozvany, Charrett, Adidam and Melchers (1969) presented a general proof for any value of α. The general procedure of the yield equality method is outlined in the next section.

3.1.1 ~~3.2~~ PROCEDURE: YIELD EQUALITY METHOD

(a) Select the order (topography) of the regions where $M_\theta = \overline{M}_\theta$ or $M_\theta = -\alpha \overline{M}_\theta$. Region boundries usually occur along concentrated line loads.

(b) In any region i, calculate the corresponding radial moments using the equilibrium equation (3.1). In the particular case of M_θ = constant & α = constant, $M'_{\theta i} = 0$, then the radial moment,

$$M_{ri} = \frac{1}{r} \int_A^r \int_A^{\bar{r}} - \bar{\bar{r}} P_i \left(\bar{\bar{r}} \right) . d\bar{\bar{r}} . d\bar{r} \quad (3.2)$$

where A is the radius of the inner boundary of the slab, $\bar{r}$ & $\bar{\bar{r}}$ are dummy variables representing radii.

(c) Determine the constants of integration and the load capacity P from the boundary, continuity and yield condition.

1. BOUNDARY CONDITIONS

(3.4)

except under a concentrated load,

$$M_r = M_\theta \tag{3.3}$$

ii. At free edges and simple discontinuous supports,

$$M_r = 0 \tag{3.4}$$

iii. At free edges with no concentrated edge loads, the radial shear Q_r is determined by integrating the equilibrium equation (3.1)

$$(rM_r)'' - M_\theta' = r.Q_r = 0$$

2. CONDITIONS OF CONTINUITY:

These refer to the continuity of radial moments and shear across a region boundary. The boundry between two adjacent regions i & (i + 1) is termed the region boundary and the radius of the region boundry is denoted by r_i.

At $r = r_i$

i. Moment Continuity:

$$M_{ri} = M_{r(i+1)} \tag{3.5}$$

ii. Shear Continuity:

$$(rM_{ri})' - M_{\theta i} = (rM_{r(i+1)})' - M_{\theta(i+1)} + Q_i \tag{3.6}$$

where Q_i is the intensity of a concentrated line load, if any, acting along the region boundary.

3. YIELD CONDITIONS

Some times it is necessary to satisfy the radial yield equalities to obtain a unique moment field, at a finite number of points along a radius of slab.

$$M_r = \overline{M}_r \quad \text{or} \quad M_r = - \beta \, \overline{M}_r \tag{3.7}$$

3.2 SOIL PRESSURE DISTRIBUTIONS

The distribution of soil pressure below footing slabs is dependent on the type of soil, water content in the soil, rigidity of the footing & eccentricity of the loading. The distribution of soil pressure below a footing slab on granular soil is very different from the one founded on cohesive soil. Hence the first assumption the structural designer must make is the general form of distribution of soil pressure. Due to simplicity, uniform pressure distribution is generally adopted. Let Q be the total load acting on a column footing of radius R. The net soil pressure distribution for various soils will be assumed as follows.

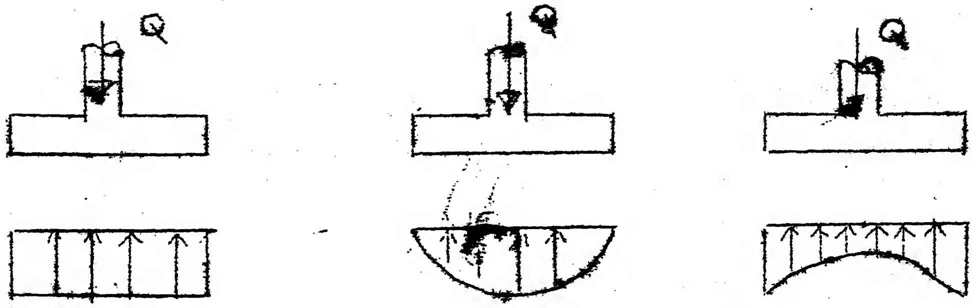
For C - ϕ soil (uniform pressure) :

$$P(r) = - \frac{Q}{\pi R^2} = - K \tag{3.8}$$

For granular soils : $P(r) = - 2K \left(1 - \frac{r^2}{r^2} \right)$ (3.9)

For cohesive soils : $P(r) = - \frac{2}{3} K \left(1 + \frac{r^2}{R^2} \right)$ (3.10)

where $K = \frac{Q}{\pi R^2}$, r is any radius from the axis of symmetry.



Uniform Soil Pressure

Granular Soil Pressure

Cohesive Soil Pressure

Fig. 3.1

3.2.1 ASSUMPTIONS

1. Behaviour of the footing slab is rigid-perfectly plastic.
2. The footing slab is "under-reinforced" (i.e. steel ratio is low), and hence the reinforcement yields prior to the crushing of concrete.
3. Membrane action is neglected.
4. The lever arm of the internal forces is constant.
5. The properties of concrete have no effect on the design.
6. The difference in effective depth of two layers of reinforcement nearer to one face of the slab is neglected.
7. Distribution of soil pressure is known.
8. The footing slab does not fail either due to diagonal tension or punching shear.

3.3 TAPERED FOOTING SLABS WITH POLAR REINFORCEMENT.

Consider a circular footing of radius R, column radius A with polar reinforcement (i.e. circular bars with equal spacing and radial bars with equal spacing along the circumference) at bottom and the depth of the footing tapered from the face of the column.

Circumferential moment capacity at the edge is M_1 while at the centre it is $M_1\ (1 + \alpha)$ where α is a factor to account for the taper of the footing slabs. The general equation of the circumferential moment capacity is then

$$M_{\theta p} = M_1\ (\ 1 + \alpha\ (1 - \frac{r}{R})) \qquad (3.11)$$

Similarly radial moment capacity at the edge is M_2. The general equation of the radial moment capacity is

$$M_{rp} = M_2\ .\ \frac{R}{r}\ (\ 1 + \alpha\ (1 - \frac{r}{R})) \qquad (3.12)$$

M_2 & M_1 are related by $M_2 = \lambda\ M_1$

$$M_{rp} = \lambda\ M_1\ \frac{R}{r}\ (\ 1 + \alpha\ (1 - \frac{r}{R})) \qquad (3.13)$$

Now, there are two failure mode shapes, viz (i) fan mechanism and (ii) the partial collapse with an outer rigid region.

Q = total load on the footing

P = uniform pressure = $Q/\pi R^2$

B = radius of the circular yield line

E = radius of the region boundary at which the value of M_θ changes.

$M_{\theta i}$ & M_{ri} are the circumferential and radial moments respectively in a region i . Introducing the non-dimensional quantities $a = A/R$, $b = B/R$, $e = E/R$, $\rho = r/R$, the moment capacities are determined as follows.

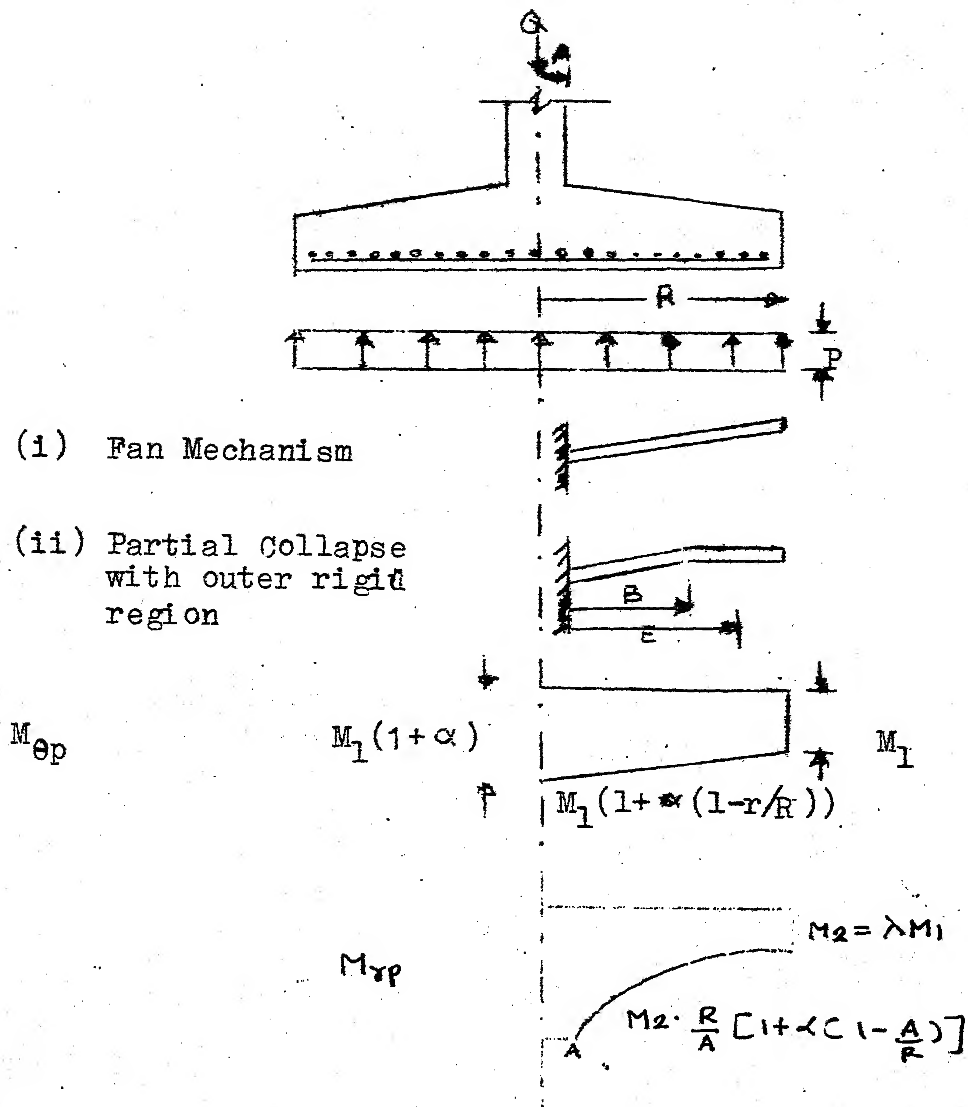


Fig. 3.2 CIRCULAR TAPERED FOOTING WITH POLER REINFORCEMENT

3.3.1 NORMAL PRESSURE DISTRIBUTION

i. FAN MECHANISM

$$(r\ M_r)'' - M_\theta' = P.r \tag{3.14}$$

$$(r\ M_r)' - M_\theta = Pr^2/2 + C_1$$

$$(r\ M_r)' = M_{\theta p} + Pr^2/2 + C_1$$

$$= M_1\ (1 + \alpha\ (1 - r/R)) + Pr^2/2 + C_1$$

$$M_r = M_1 (1 + \alpha) - M_1 \alpha\, r/2R + Pr^2/6 + C_1 + C_2/r \qquad (3.15)$$

The Boundary and Continuity Conditions are

$$\left.\begin{aligned} M_r &= M_2 (R/A) \left(1+\alpha(1 - A/R)\right) && \text{at } r = A \\ M_r &= 0 && \text{at } r = R \\ (r\, M_r)' - M_\theta &= 0 && \text{at } r = R \end{aligned}\right| \qquad (3.16)$$

Finally

$$\frac{M_1}{PR^2} = \frac{(2-a(1+a))\,(1-a)}{6(1-a)(1+\lambda\alpha) + 3\alpha(1-a)^2+6\lambda} \qquad (3.17)$$

ii. PARTIAL COLLAPSE WITH OUTER RIGID REGION

The region topography is given in Fig. 3.2

$$\begin{aligned} M_{\theta 1} &= M_1(1+\alpha(1- r/R)) && ;\ A \leq r \leq E \\ M_{\theta 2} &= 0 && ;\ E \leq r \leq R \end{aligned} \qquad (3.18)$$

where E is the radius of the region boundary at which the value of M_θ changes. The boundary and continuity ~~and~~ conditions are given by

$$\begin{aligned}
&\text{i.}\ M_{r1} = M_2 \frac{R}{A} \left(1+\alpha(1-A/R)\right) && \text{at } r = A \\
&\text{ii.}\ M_{r1} = 0 && \text{at } r = B \\
&\text{iii.}\ M_{r1}' = 0 && \text{at } r = B \\
&\text{iv.}\ M_{r1} = M_{r2} && \text{at } r = E \qquad (3.19) \\
&\text{v.}\ (r\, M_{r1})' - M_{\theta 1} = (r\, M_{r2})' - M_{\theta 2} && \text{at } r = E \\
&\text{vi.}\ (M_{r2}) = 0 && \text{at } r = R \\
&\text{vii.}\ (r\, M_{r2})' = 0 && \text{at } r = R
\end{aligned}$$

(3.10)

$$M_{r1} = M_1(1+\alpha) - M_1\alpha\frac{r}{2R} + \frac{Pr^2}{6} + C_{11} + C_{21}/r$$

(3.20)

$$M_{r2} = Pr^2/6 + C_{12} + C_{22}/r$$

Finally

$$\frac{M_1}{PR^2} = \frac{1-b^2}{2(1+\alpha(1-b))} \tag{3.21}$$

and equation for b is

$$(\alpha)\, b^4 + ((-4(1+\alpha))\, b^3 + (\alpha(a(6\lambda+6-3a)-6\lambda+3) + 6a - 6\lambda)\, b^2$$

$$+ (2a\alpha(a^2-3))b+((1+\alpha)(6\lambda-2a^3)+3a\alpha(a-2\lambda)) = 0 \tag{3.22}$$

3.3.2 GRANULAR SOIL: PRESSURE DISTRIBUTION

$$P(r) = 2K(1-r^2/R^2) \quad \text{where } K = \frac{Q}{\pi R^2} \tag{3.23}$$

FAN MECHANISM

$$\frac{M_1}{KR^2} = \frac{(3a^5-10a^3+15a-8)}{(15((1+\alpha)(2a-2\lambda-1)+\alpha a(2\lambda-a)-1))} \tag{3.24}$$

PARTIAL COLLAPSE MECHANISM

$$\frac{M_1}{KR^2} = \frac{(1-b^2)^2}{(2(1+\alpha(1-b)))} \tag{3.25}$$

(3.11)

Equation for b is

$$(-9\alpha)\, b^6 + (24(1+\alpha))\, b^5 + (10\alpha + 30(1+\alpha)(\lambda - a) + 15\alpha\, a(a - 2\lambda))b^4 + (-40(1+\alpha))b^3$$
$$+ (15\alpha - 60(1+\alpha)(\lambda - a) - 30\alpha\, a(a - 2\lambda))b^2$$
$$+ (-\alpha(30a - 20a^3 + 6a^5))\, b + ((1+\alpha)(30a - 20a^3 + 6a^5) + 30(1+\alpha)(\lambda - a) + 15\alpha\, a(a - 2\lambda)) = 0 \qquad (3.26)$$

Equation for e is

$$e^2 - (2(1+\alpha)/\alpha)\, e + ((-15a + 10a^3 - 3a^5 + 8)/(15\alpha(M_1/Kr^2)) + (2(1+\alpha)(a - \lambda) + \alpha a(2\lambda - a))/\alpha) = 0 \qquad 3.27)$$

3.3.3 COHESIVE SOIL PRESSURE DISTRIBUTION

$$P(r) = \frac{2}{3} K\,(1 + r^2/R^2) \text{ where } K = \frac{Q}{\pi R^2} \qquad (3.28)$$

i. FAN MECHANISM

$$\frac{M_1}{KR^2} = \frac{(45a - 10a^3 - 3a^5 - 32)}{90\left[(a - \lambda - 1)(1+\alpha) - \frac{\alpha}{2}(a^2 - 2\lambda a - 1)\right]} \qquad (3.29)$$

ii. PARTIAL COLLAPSE

$$M_1/KR^2 = (MPRP) = 2\left[1 - \left(\frac{1+b^2}{2}\right)^2\right] / (3(1+\alpha(1-b))) \qquad (3.30)$$

Equation for b is

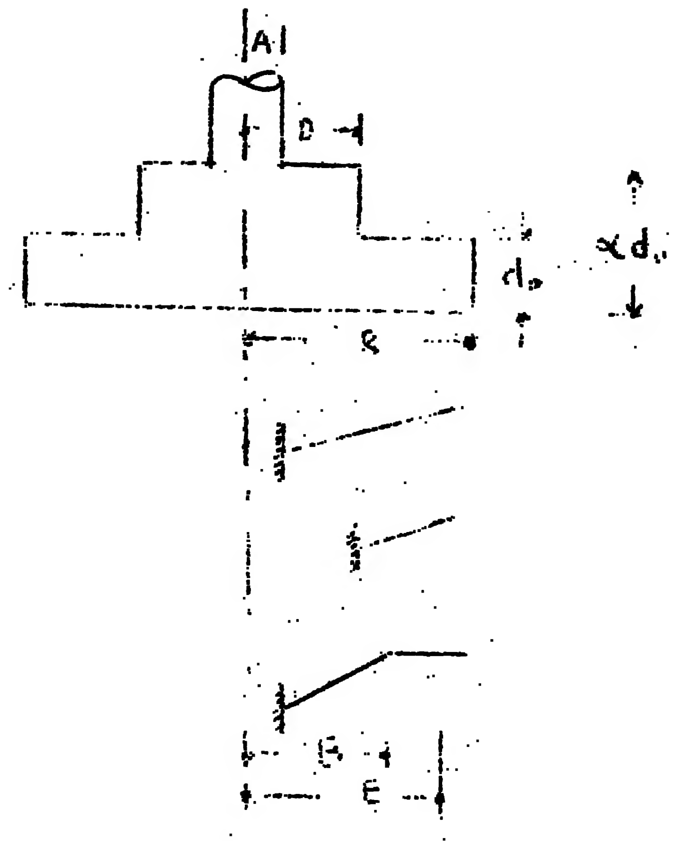
$$(-9\alpha)b^6 + (24\,(1+\alpha))b^5 + \Big[-10\alpha + 30(1+\alpha)(\lambda - a) + 15\alpha a\,(a - 2\lambda)\Big]\, b^4 + (40\,(1+\alpha))b^3 + \Big[-45\alpha + 60(1+\alpha)(\lambda - a) + 30\alpha a(a - 2\lambda)\Big]\, b^2 + (2\alpha\,(45a - 10a^3 - 3a^5))b + \Big[-2\,(1+\alpha)\,(45\lambda - 10a^3 - 3a^5) - 45\alpha a(a - 2\lambda)\Big] = 0 \tag{3.31}$$

Equation for e is

$$e^2 - (2\,(1+\alpha)/\alpha)e - \Big[2(1+\alpha)\,(\lambda - a) + \alpha a(a - 2\lambda) + (-32 + 45a - 10a^3 - 3a^5)/(90 \,.\, MPRP)\Big]/\alpha = 0 \tag{3.32}$$

3.4 STEPPED FOOTING SLABS

If radius of footing is large, or sometimes due to ease of construction, instead of tapered footing, stepped footing is used. Let D be radius of the step. Thickness of the footing is changed by factor α. $d = D/R$ is used as non dimensional~~ise~~ parameter. Here we will analyse for 2 types of reinforcement i) isotopic ii. polar. In this case there are 2 possible fan mechanisms. One at the face of the column and second at the change of thickness. In case of partial collapse both B and E may be inside the step or outside step ($b < d$ or $b > d$ and $e < d$ or $e > d$)



3.3 STEPPED FOOTING WITH ISOTROPIC REINFORCEMENT

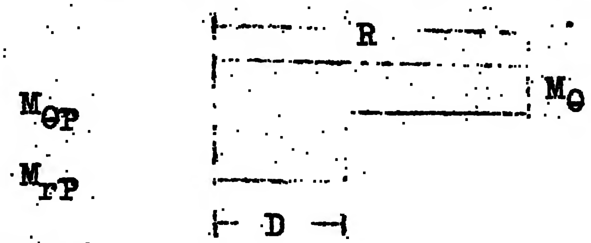


FIG 3.4

3.5.1 GRANULAR SOIL PRESSURE DISTRIBUTION

1. For Fan Mechanism there are 2 regions

1. $0 \leq r \leq D$ 2. $D \leq r \leq R$

$M_\theta = \alpha M_o$ $M_\theta = M_o$

$$(r M_r)'' - M_\theta' = P(r) \cdot r \tag{3.33}$$

$$P(r) = 2K\left(1 - \frac{r^2}{R^2}\right) \quad K = Q/\pi R^2$$

$$(r M_r)' - M_\theta = 2K\left(r^2/2 - \frac{r^4}{4R^2}\right) + C_1 \tag{3.34}$$

REGION 1

$$M_{r1} = \alpha M_o + K\left(r^2/3 - \frac{r^4}{10R^2}\right) + C_{11} + C_{21}/r$$
$$M_{r2} = M_o + K\left(r^2/3 - \frac{r^4}{10R^2}\right) + C_{12} + C_{22}/r \qquad (3.35)$$

B.C. and C.C. are

1. $M_{r1} = \alpha M_o$ at $r = A$
2. $M_{r1} = M_{r2}$ at $r = D$
3. $(rM_{r1})' - M_{\theta 1} = (r\, M_{r2})' - M_{\theta 2}$ at $r = D$ (3.36)
4. $M_{r2} = 0$ at $r = R$
5. $(r\, M_{r2})' - M_{\theta 2} = 0$ at $r = R$

Finally

$$\frac{M_o}{KR^2} = (8 - 15\, a + 10\, a^3 - 3\, a^5)/\, 30\, (\alpha d + 1) \qquad (3.37)$$

ii. PARTIAL COLLAPSE

Considering $B > D$ and $E > D$ case, there are 3 regions

i. $A \leq r \leq D$; $M_{\theta 1} = \alpha M_o$ ii. $D \leq r \leq E$; $M_{\theta 2} = M_o$

iii. $E \leq r \leq R$; $M_{\theta 3} = 0$

$$M_{r1} = \alpha M_o + K\left(\frac{r^2}{3} - \frac{r^4}{10R^2}\right) + C_{11} + C_{21}/r$$

(3.15)

$$M_{r2} = M_o + K\left(\frac{r^2}{3} - \frac{r^4}{10R^2}\right) + C_{12} + \frac{C_{22}}{r}$$

$$M_{r3} = K\left(\frac{r^2}{3} - \frac{r^4}{10R^2}\right) + C_{13} + \frac{C_{23}}{r} \qquad (3.38)$$

B.C. and C.C. are

1. $M_{r1} = \propto M_o$ at $r = A$
2. $M_{r2} = 0$ at $r = B$
3. $(M_{r2})' = 0$ at $r = B$
4. $(M_{r1}) = M_{r2}$ at $r = D$
5. $(r\, M_{r1})' - M_{\theta 1} = (r\, M_{r2})' - M_{\theta 2}$ at $r = D$ (3.39)
6. $M_{r2} = M_{r3}$ at $r = E$
7. $(r\, M_{r2})' - M_{\theta 2} = (r\, M_{r3})' - M_{\theta 3}$ at $r = E$
8. $M_{r3} = 0$ at $r = R$
9. $(r\, M_{r3})' = 0$ at $r = R$

Finally

$$\frac{M_o}{KR^2} = (1 - b^2)^2/2 = (\text{MPRP}) \qquad (3.40)$$

Equation for b , b > d

$$(-12)\, b^5 + (15d\ (1 - \propto))\, b^4 + (20)\, b^3 + (-30d(1 - \propto))b^2$$
$$+ (-15a + 10a^3 - 3a^5 + 15d\ (1 - \propto)) = 0 \qquad (3.41)$$

(3.16)

$$e = \left\{\frac{8 - 15a + 10a^3 - 3a^5}{30 \cdot (MPRP)}\right\} + d(1 - \alpha)$$

valid for $e > d$. (3.42)

In case $b < d$

$$\frac{M_o}{KR^2} = \frac{(1-b^2)^2}{2\alpha} = (MPRP) \quad (3.43)$$

Equation for b is

$$(12)b^5 + (-20)b^3 + (3a^5 - 10a^3 + 15a) = 0 \quad (3.44)$$

in case $e < d$ $(b < e \; ; \; b < d)$

$$e = \frac{(8 - 15a + 10a^3 - 3a^5)}{60\alpha \cdot (MPRP)} \quad (3.45)$$

We observe that e for $e < d$ is exactly half of $e > d$

3.5.2 COHESIVE SOIL PRESSURE DISTRIBUTION

$$P(r) = \frac{2K}{3} (1 + r^2/R^2) \quad (3.46)$$

Fan mechanism at A

$$\frac{M_o}{KR^2} = \frac{(32 - 45a + 10a^3 + 3a^5)}{90 (1 + d(\alpha - 1))} \quad (3.47)$$

Fan mechanism at D

$$M_o / KR^2 = (32 - 45d + 10d^3 + 3d^5) / 90 \quad (3.48)$$

Partial Collapse Mechanism

for $b < d$

$$\frac{M_o}{KR^2} = \frac{2}{3} \left(1 - \left\{\frac{1+b^2}{2}\right\}^2\right) / \alpha = (MPRP) \quad (3.49)$$

Equation for b

$$(12)b^5 + (20)b^3 + (3a^5 + 10a^3 - 45a) = 0 \qquad (3.50)$$

for $b > d$

$$\frac{M_o}{KR^2} = \frac{2}{3}\left(1 - \left(\frac{1 + b^2}{2}\right)^2 \right) = (MPRP) \qquad (3.51)$$

Equation for b

$$(12)\, b^5 + (15d\,(\alpha - 1))b^4 + (20)b^3 + (30d(\alpha - 1))b^2 + (3a^5 + 10a^3 - 45a + 45d\,(1 - \alpha)) = 0 \qquad (3.52)$$

for $e < d$

$$e = \frac{(32 - 45a + 10a^3 + 3a^5)}{90\alpha\ (MPRP)} \qquad (3.53)$$

for $e > d$

$$e = \frac{(32 - 45a + 10a^3 + 3a^5)}{90\ .\ (MPRP)} - d\ (\alpha - 1) \qquad (3.54)$$

3.5.3 UNIFORM PRESSURE DISTRIBUTION

$$P\ (r) = \frac{Q}{\pi R^2} \qquad (3.55)$$

Fan mechanism at A

$$\frac{M_o}{PR^2} = \frac{(2-a-a^2)\ (1-a)}{6(1+d\ (\alpha - 1))} \qquad (3.56)$$

Fan mechanism at D

$$\frac{M_o}{PR^2} = \frac{(2-d-d^2)\ (1-d)}{6} \qquad (3.57)$$

Partial Collapse Mechanism

for $b < d$

$$\frac{M_o}{PR^2} = (1-b^2)/\,2\alpha = (MPRP) \tag{3.58}$$

equation for b is

$$b = \left[\frac{(3-a^2)a}{2}\right]^{1/3} \tag{3.59}$$

for $b > d$

$$\frac{M_o}{PR^2} = \frac{1-b^2}{2} \tag{3.60}$$

equation for b is

$$(2)b^3 + (3(\alpha-1)d)\,b^2 + (a^3 - 3(\alpha-1)d - 3a) = 0 \tag{3.61}$$

for $e < d$

$$e = \frac{2}{3}\left\{\frac{(b^2+b+1)}{(b+1)}\right\} = \frac{(1-b^3)}{3\alpha(MPRP)} \tag{3.62}$$

for $e > d$

$$e = \frac{2\alpha}{3}\left\{\frac{1+b+b^2}{1+b}\right\} - d\,(\alpha-1) \tag{3.63}$$

3.6 STEPPED FOOTING WITH POLAR REINFORCEMENT

3.6.1 COHESIVE SOIL PRESSURE DISTRIBUTION

$$P(r) = \frac{2}{3}\frac{Q}{\pi R^2}\left\{1 + \frac{r^2}{R^2}\right\} = \frac{2}{3}K\left\{1 + \frac{r^2}{R^2}\right\} \tag{3.65}$$

(3.19)

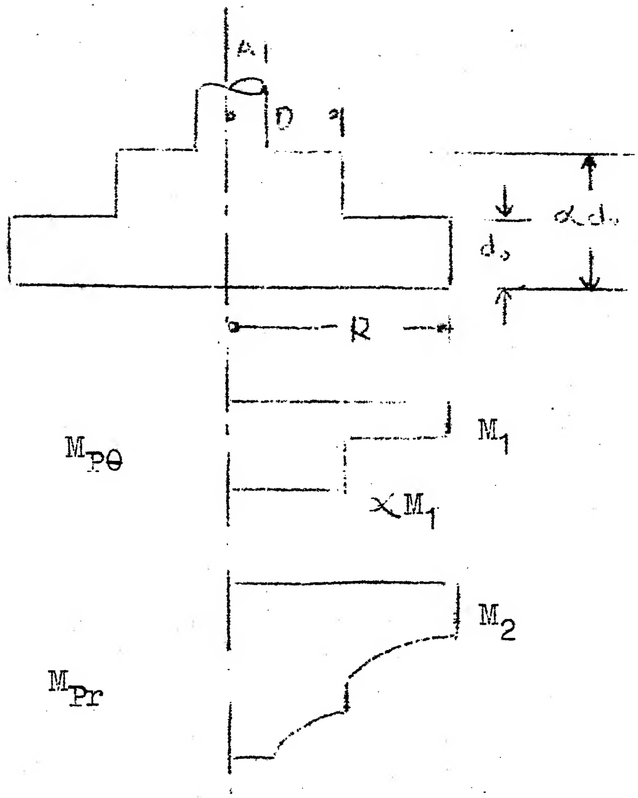


FIG 3.5

FAN MECHANISM

Region 1 $\quad A \leq r \leq D \quad M_{\theta 1} = \alpha M_1$

Region 2 $\quad D \leq r \leq R \quad M_{\theta 2} = M_1$

Equilibrium Equation is

$$(r\,M_r)'' - M_\theta' = \frac{2}{3} K \left\{ 1 + \frac{r^2}{R^2} \right\} \cdot r$$

Integrating

$$M_{r1} = \alpha M_1 + K \left\{ \frac{r^2}{9} + \frac{r^4}{30R^2} \right\} + C_{11} + C_{21}/r \qquad (3.66)$$

$$M_{r2} = M_1 + K \left\{ \frac{r^2}{9} + \frac{r^4}{30R^2} \right\} + C_{12} + C_{22}/r$$

Now Boundary and Continuity Conditions are

i. $\quad M_{r1} = \dfrac{\lambda \alpha M_1}{a} \quad$ at $r = A$

ii. $\quad M_{r1} = M_{r2} \quad$ at $r = D$

(3.20)

iii. $(r M_{r1})' - M_{\theta 1} = (r M_{r2})' - M_{\theta 2}$ at $r = D$

iv. $M_{r2} = 0$ at $r = R$ (3.67)

v. $(r M_{r2})' - M_{\theta 2} = 0$ at $r = R$

Finally

$$\frac{M_1}{KR^2} = \frac{32 - 45a + 10a^3 + 3a^5}{90 \{ \alpha(d+\lambda-a) + 1-d \}} \tag{3.68}$$

FAN MECHANISM AT THE STEP

Substituting $a = d$ and $\alpha = 1$

$$\frac{M_1}{KR^2} = \frac{32 - 45d + 10d^3 + 3d^5}{90 (1 + \lambda - d)} \tag{3.69}$$

PARTIAL COLLAPSE MECHANISM

In case of $B < D$ and $E > D$

Region 1 $A \leq r \leq D$ $M_{\theta 1} = \alpha M_1$

Region 2 $D \leq r \leq E$ $M_{\theta 2} = M_1$ (3.70)

Region 3 $E \leq r \leq R$ $M_{\theta 3} = 0$

$$M_{r1} = \alpha M_1 + K \left\{ \frac{r^2}{9} + \frac{r^4}{30R^2} \right\} + C_{11} + \frac{C_{21}}{r}$$

$$M_{r2} = M_1 + K \left\{ \frac{r^2}{9} + \frac{r^4}{30R^2} \right\} + C_{12} + \frac{C_{22}}{r}$$

$$M_{r3} = \cdot K \left\{ \frac{r^2}{9} + \frac{r^4}{30R^2} \right\} + C_{13} + \frac{C_{23}}{r}$$

B .C. and C.C. are

i. $M_{r1} = \dfrac{\lambda M_1 \alpha}{a}$ at $r = A$

ii. $(M_{r1}) = 0$ at $r = B$

iii. $(M_{r1})' = 0$ at $r = B$

iv. $M_{r1} = M_{r2}$ at $r = D$

v. $(r\, M_{r1})' - M_{\theta 1} = (r\, M_{r2})' - M_{\theta 2}$ at $r = D$ (3.71)

vi. $M_{r2} = M_{r3}$ at $r = E$

vii. $(r\, M_{r2})' - M_{\theta 2} = (r\, M_{r3})' - M_{\theta 3}$ at $r = E$

viii. $(M_{r3}) = 0$ at $r = R$

ix. $(r\, M_{r3})' = 0$ at $r = R$

Finally, for $B < D$, $E > D$

$$\frac{M_1}{KR^2} = \frac{\frac{2}{3}\left\{ 1 - \left\{\frac{1+b^2}{2}\right\}^2 \right\}}{\alpha} = (MPRP) \tag{3.72}$$

Equation for b is

$$(-12)\, b^5 + (-15\, (\lambda - a)\ )\, b^4 + (-20)\, b^3 + (-30\, (\lambda - a)\,)b^2 + 45\, (\lambda - a) - 10a^3 - 3a^5 + 45a = 0 \tag{3.73}$$

$$e = \frac{32 - 45a + 10a^3 + 3a^5}{90\, (MPRP\ )} - \left\{ \alpha\, (d + \lambda - a) - d \right\} \tag{3.74}$$

(3.22)

For $E < D$

$$e = \frac{32 - 45a + 10a^3 + 3a^5}{90 . \alpha . (MPRP)} - (\lambda - a) \tag{3.75}$$

For $B > D$

$$\frac{M_1}{KR^2} = \frac{2}{3} \left\{ 1 - \left(\frac{1+b^2}{2}\right) \right\}^2 = (MPRP) \tag{3.76}$$

Equation for b is

$$(-12)\, b^5 + (-15 \ (d+\lambda-a) + 15d)\, b^4 + (-20)\, b^3$$

$$+ (-30\alpha(d+\lambda-a) + 30d)\, b^2 + (45\alpha(d+\lambda-a) - 45d + 45a - 10a^3 - 3a^5)$$

$$= 0 \tag{3.77}$$

3.6.2 GRANULAR SOIL PRESSURE DISTRIBUTION

$$P(r) = 2K\left(1 - \frac{r^2}{R^2}\right) \tag{3.78}$$

FAN MECHANISM AT A

$$\frac{M_1}{KR^2} = \frac{8 - 15a + 10a^3 - 3a^5}{30\,(\alpha(d+\lambda-a) + 1 - d)} \tag{3.79}$$

FAN MECHANISM AT D

$$\frac{M_1}{KR^2} = \frac{8 - 15d + 10d^3 - 3d^5}{30(\lambda + 1 - d)} \tag{3.80}$$

PARTIAL COLLAPSE MECHANISM

For $B < D$

$$\frac{M_1}{KR^2} = \frac{(1-b^2)^2}{2\alpha} = (MPRP) \tag{3.81}$$

Equation for b is

$$(-12)b^5 + (15(a-\lambda))b^4 + (20)b^3 + (30(\lambda-a))b^2 + (10a^3 - 3a^5 - 15a + 15(a-\lambda)) = 0 \qquad (3.82)$$

For B > D

$$\frac{M_1}{KR^2} = \left\{\frac{1-b^2}{2}\right\}^2 = (MPRP) \qquad (3.83)$$

Equation for b is

$$(12)b^5 + \{15\alpha(d+\lambda-a) - 15d\}b^4 + (-20)b^3 + \{-30\alpha(d+\lambda-a) + 30d\}b^2 + \{15\alpha(d+\lambda-a) - 15d + 15a - 10a^3 + 3a^5\} = 0 \qquad (3.84)$$

for E < D

$$e = \frac{8-15a+10a^3-3a^5}{30\alpha(MPRP)} + (a-\lambda) \qquad (3.85)$$

For E > D

$$e = \frac{8-15a+10a^3-3a^5}{30\,(MPRP)} - \{\alpha(\lambda-a) + (\alpha-1)d\} \qquad (3.86)$$

3.6.3 UNIFORM PRESSURE DISTRIBUTION

$$P(r) = \frac{Q}{\pi R^2} \qquad (3.87)$$

FAN MECHANISM AT A

$$\frac{M_1}{PR^2} = \frac{(a^2+a-2)(1-a)}{6\{\alpha(a-\lambda-d)+d-1\}} \qquad (3.88)$$

FAN MECHANISM AT D

$$\frac{M_1}{PR^2} = \frac{(2-3a + a^3)}{6\,(\lambda + 1 - a)} \tag{3.89}$$

PARTIAL COLLAPSE MECHANISM

For $b < d$

$$\frac{M_1}{PR^2} = \frac{1-b^2}{2\alpha} = (MPRP) \tag{3.90}$$

Equation for b is

$$(4\alpha)b^3 + (6\alpha(\lambda - a))b^2 + (2\alpha\,(a^3 - 3\lambda)) = 0 \tag{3.91}$$

For $b > d$

$$\frac{M_1}{PR^2} = \frac{1-b^2}{2} = (MPRP) \tag{3.92}$$

Equation for b is

$$\begin{aligned}(-2)b^3 &+ (3\,(a-\lambda-d)\,\alpha + 3d\,)b^2 \\ &+ (3a - a^3 - 3(a-\lambda-d)\alpha - 3d) = 0\end{aligned} \tag{3.93}$$

For $E < D$

$$e = \frac{2 - 3a + a^3}{6\alpha\,(MPRP)} + (a - \lambda) \tag{3.94}$$

For $E > D$

$$e = \frac{2 - 3a + a^3}{6\,(MPRP)} + (\alpha(a - \lambda - d) + d) \tag{3.95}$$

3.7.1 COMPUTER PROGRAMS

Computer program was written in Fortran IV language and problems were solved partly on our IBM 7044 computer at IIT Kanpur & on CDC 3600 at TIFR. at Bombay. To get value of b , polynomial equations were solved by Newton-Raphson procedure.

In case of Tapered Footings, & Stepped footings with polar reinforcement ; λ was assumed as 0.15 for preparation of design charts.

In case of Stepped footing , the radius of the step, D was assumed such that $d = D/R = 0.5$

3.7.2 DESIGN EXAMPLE

Following example will illustrate the use of Design charts.

Data : 1) Column Radius $A = 15$ cm.

2) Radius of footing slab $R = 100$ cm

3) $a = A/R = 0.15$

4) Load on the column $Q = 45000$ Kg.

5) Uniform pressure $P = \dfrac{Q}{\pi R^2} = 1.43$ Kg/cm^2

For a Tapered footing with polar reinforcement let $\alpha = 1$. From Design chart No.1 we get

$$\frac{M_o}{PR^2} = 0.19$$

$$M_o = 0.19 \times 1.43 \times (100)^2$$

$$= 2720 \text{ Kg-cm/cm}$$

Circumferential moment capacity to be provided is

$$M_o = 2720 \text{ Kg-cm/cm}$$

Now moment capacity can be written as follows

$$M_p = \frac{0.95\ d\ F_y\ A_{st}}{S}$$

where , M_p = Moment capacity/unit width

d = effective depth of the footing

S = spacing of the reinforcement

F_y = yield stress for steel

A_{st} = Area of cross section of steel bar

Equating $M_o = M_p$ we get

$$2720 = \frac{0.95\ d\ F_y\ A_{st}}{S}$$

Let effective depth of the footing at end be equal to

$$d = 0.75\ d_o$$

where , d_o = Total depth of footing at the end

Let $S = 10$ cm.

For Mild steel $F_y = 2600\ Kg/cm^2$

$$d = \frac{2720 \times 10}{0.95 \times 2600 \times A_{st}} = \frac{11}{A_{st}}$$

Assuming 12 mm diameter bars

$$A_{st} = 1.13\ cm^2$$

$$d = 9.73\ cm$$

$$d_o = 13\ cm$$

Thus 13 cm depth should be provided at end while at $R = 0$ total depth will be 26 cm.

Since $M_2 = \lambda M_o$

λ is assumed as 0.15

$M_2 = 0.15 \times 2720$

$= 408$ Kg-cm/cm

$S = 66.1$ cm

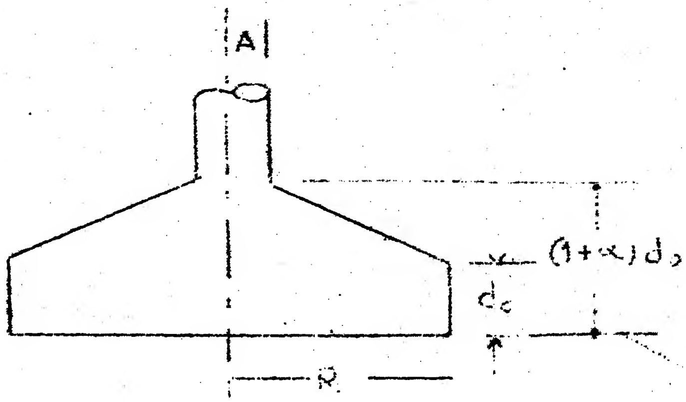
Total cirumference of the footing $= \pi R$

$= 314$ cm

Total no. of radial bars to be provided $= \frac{314}{66.1} \simeq 5$

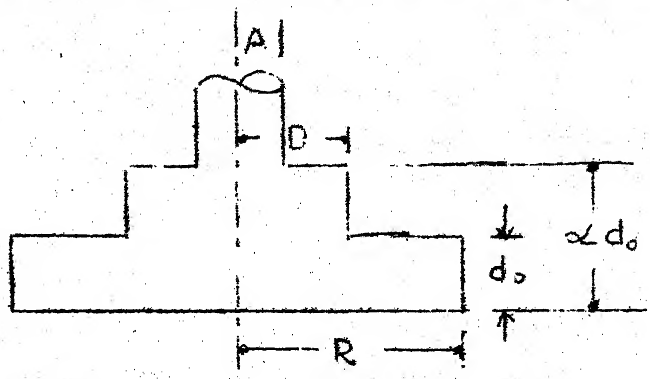
Thus provide 12 mm diameter circular rings at 10 cm C/C and 5 radial bars of same diameter .

$M_2 = \lambda M_1$
$\lambda = 0.15$



TAPERED FOOTING WITH POLAR REINFORCEMENT

$M_2 = \lambda M_1$
$\lambda = 0.15$

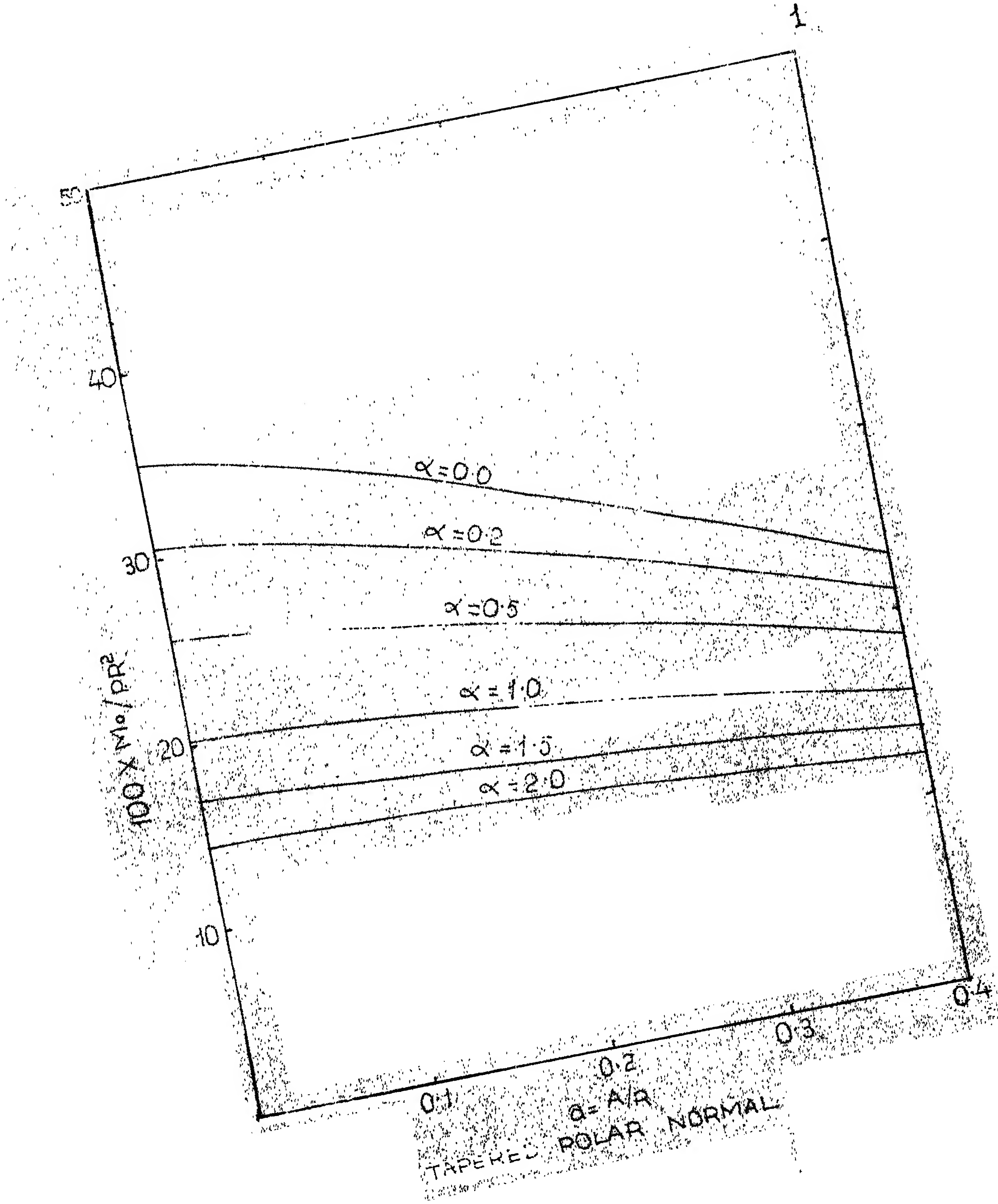


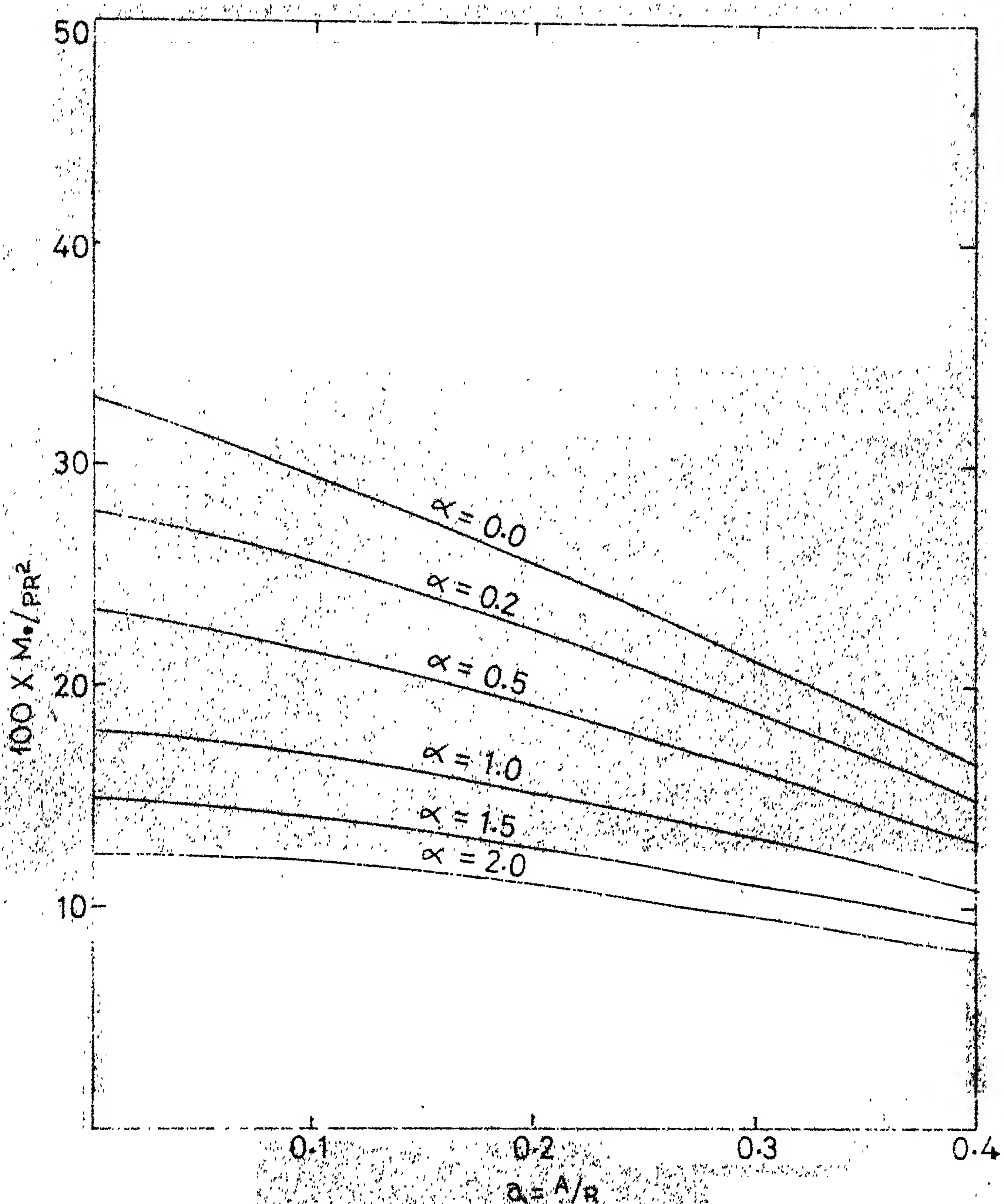
STEPPED FOOTINGS

For polar net reinforcements

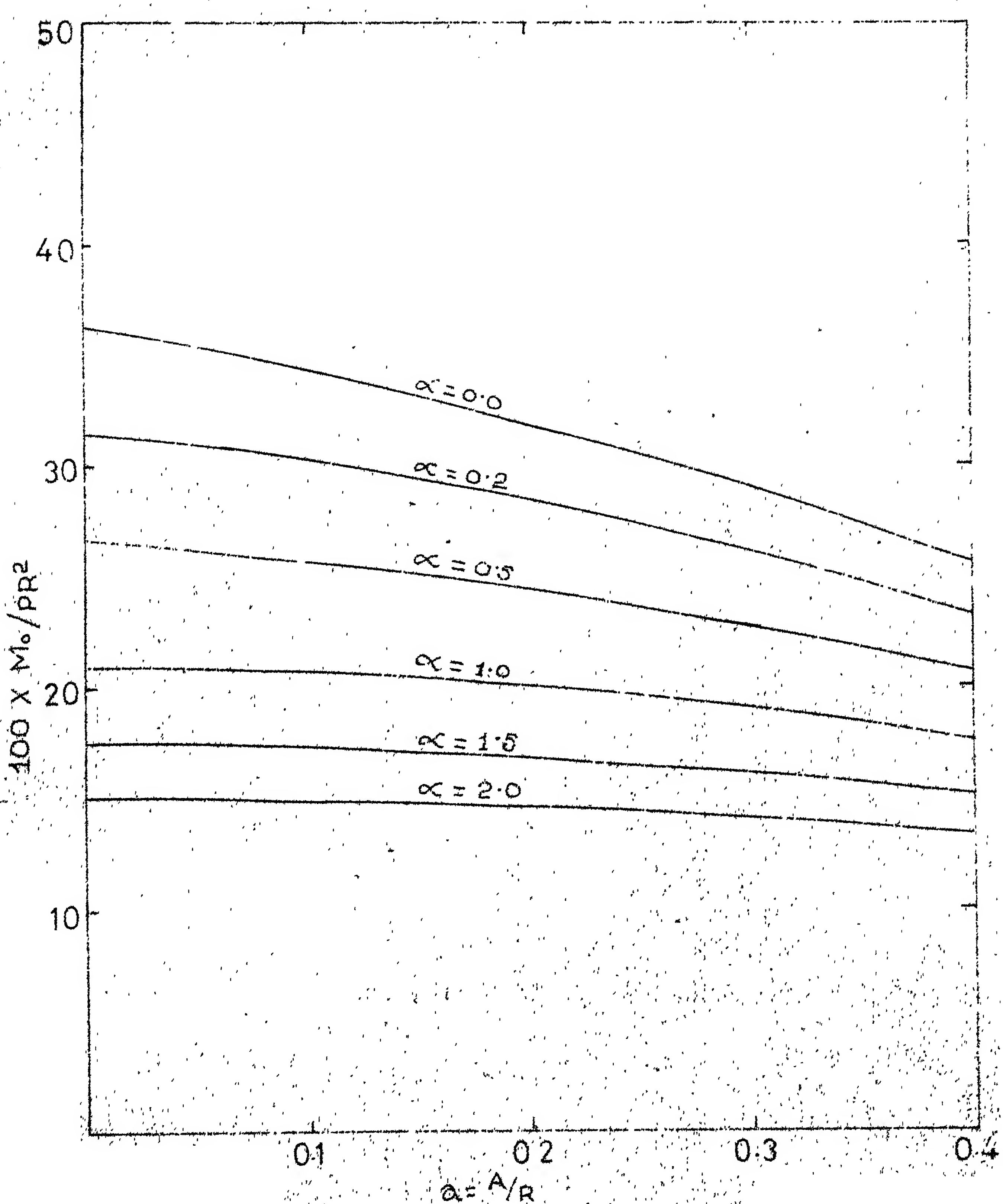
value of $M_2 = \lambda M_1$
$\lambda = 0.15$

M_o or M_1 are circumferential moment capacity to be provided at the end

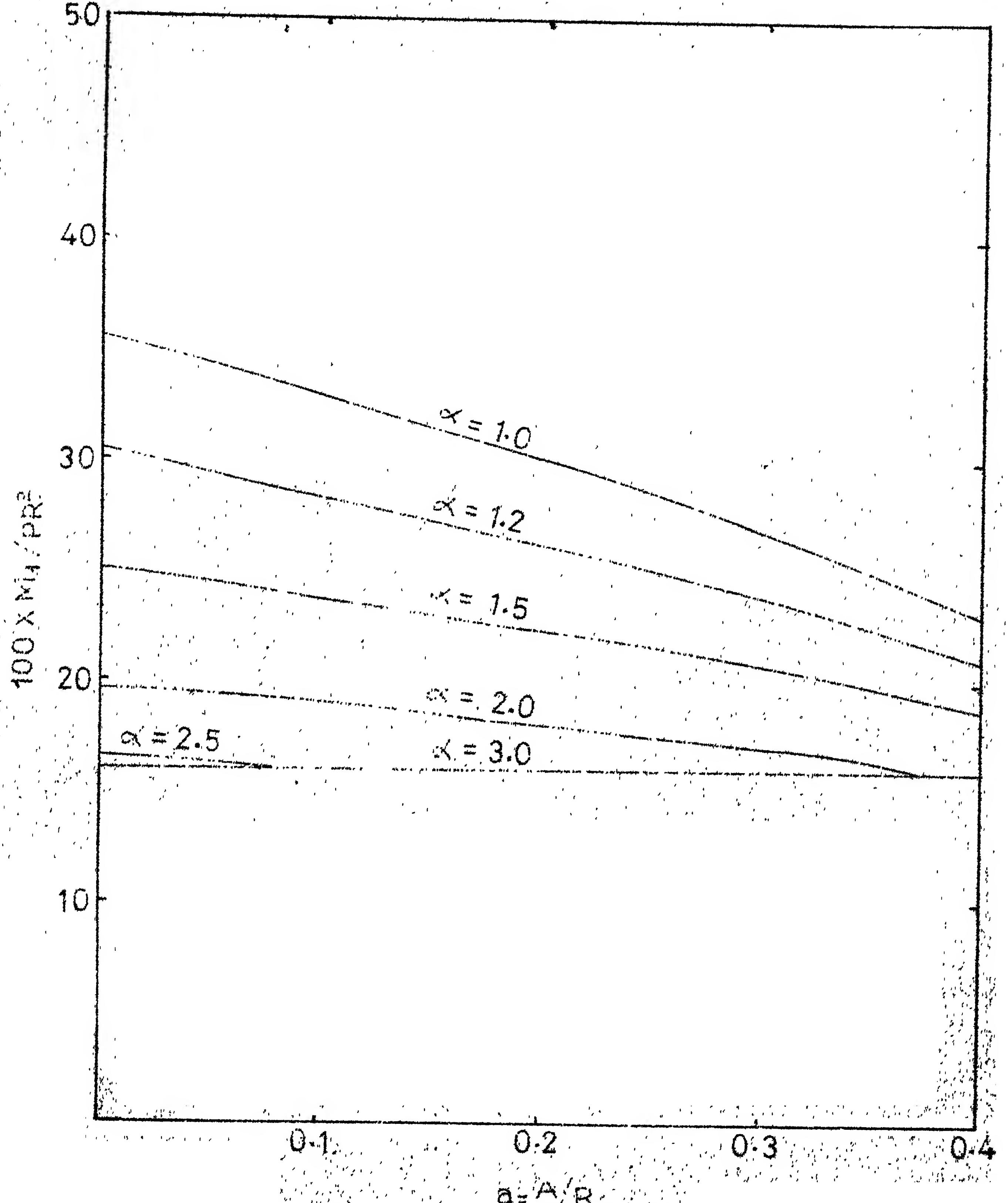

1
50
40
30
20
10
100 X Mo / pR²
α=0·0
α=0·2
α=0·5
α=1·0
α=1·5
α=2·0
0·1
0·2
0·3
0·4
a= A/R
TAPERED POLAR NORMAL



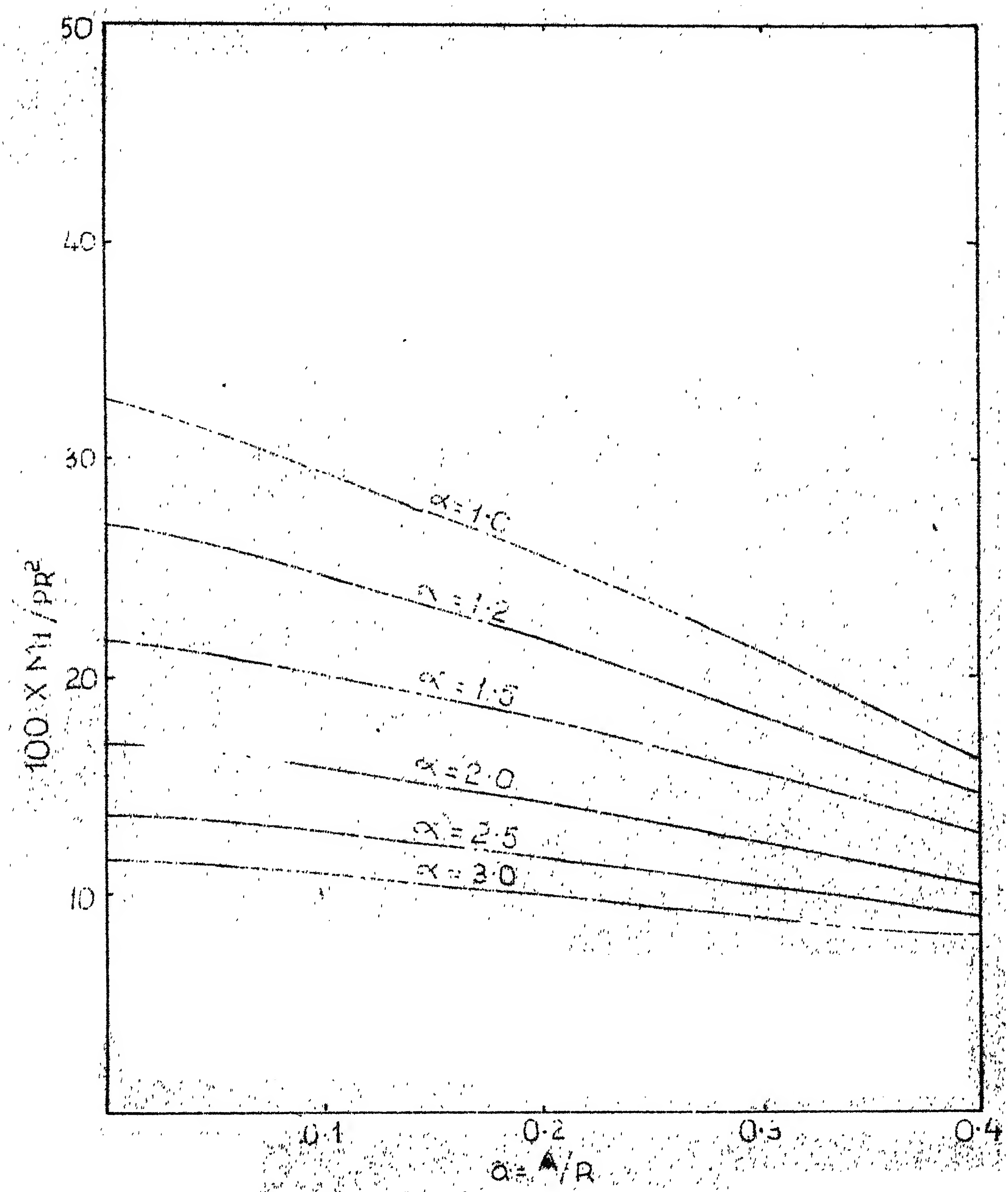
TAPERED POLAR GRANULAR



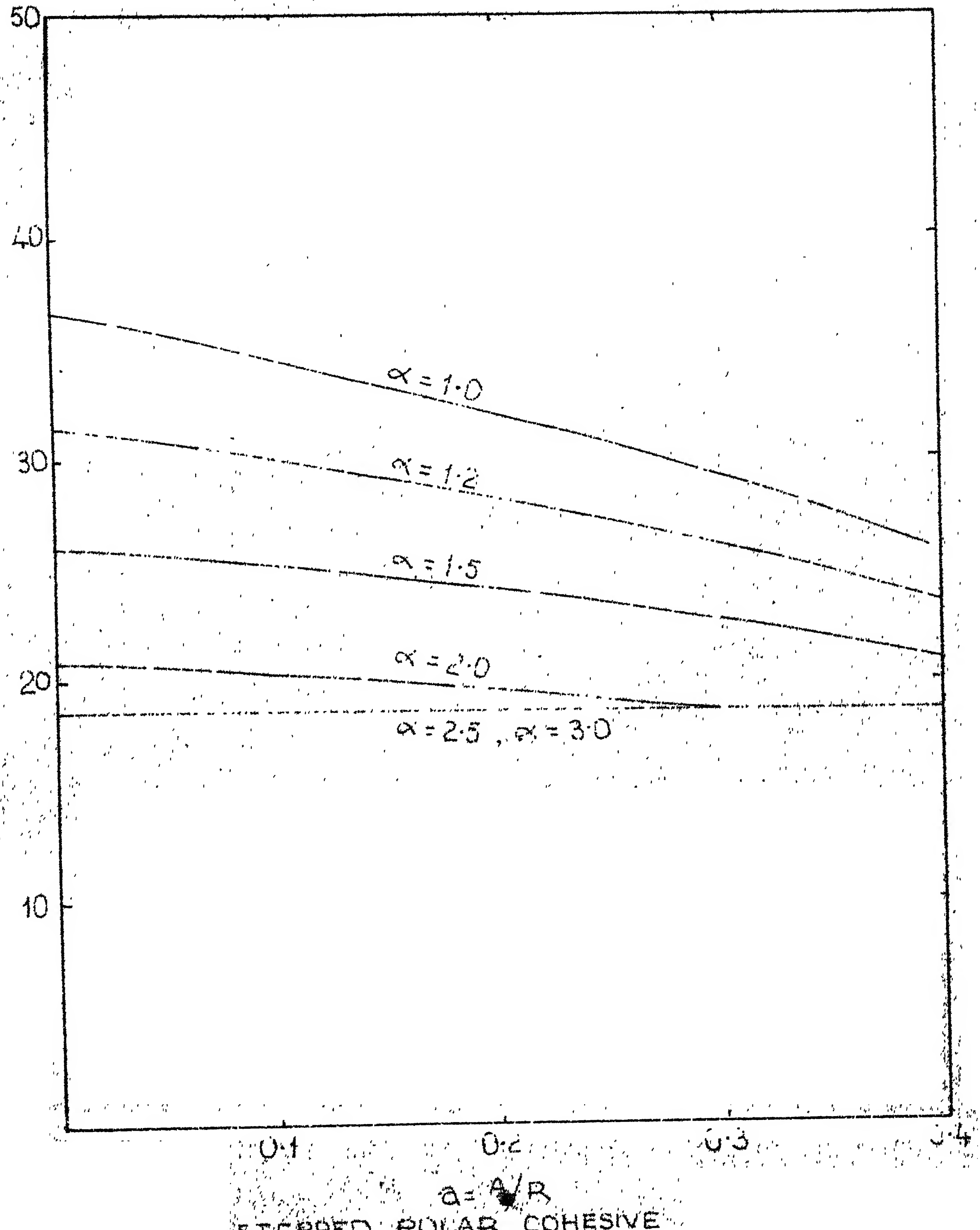
TAPERED POLAR COHESIVE



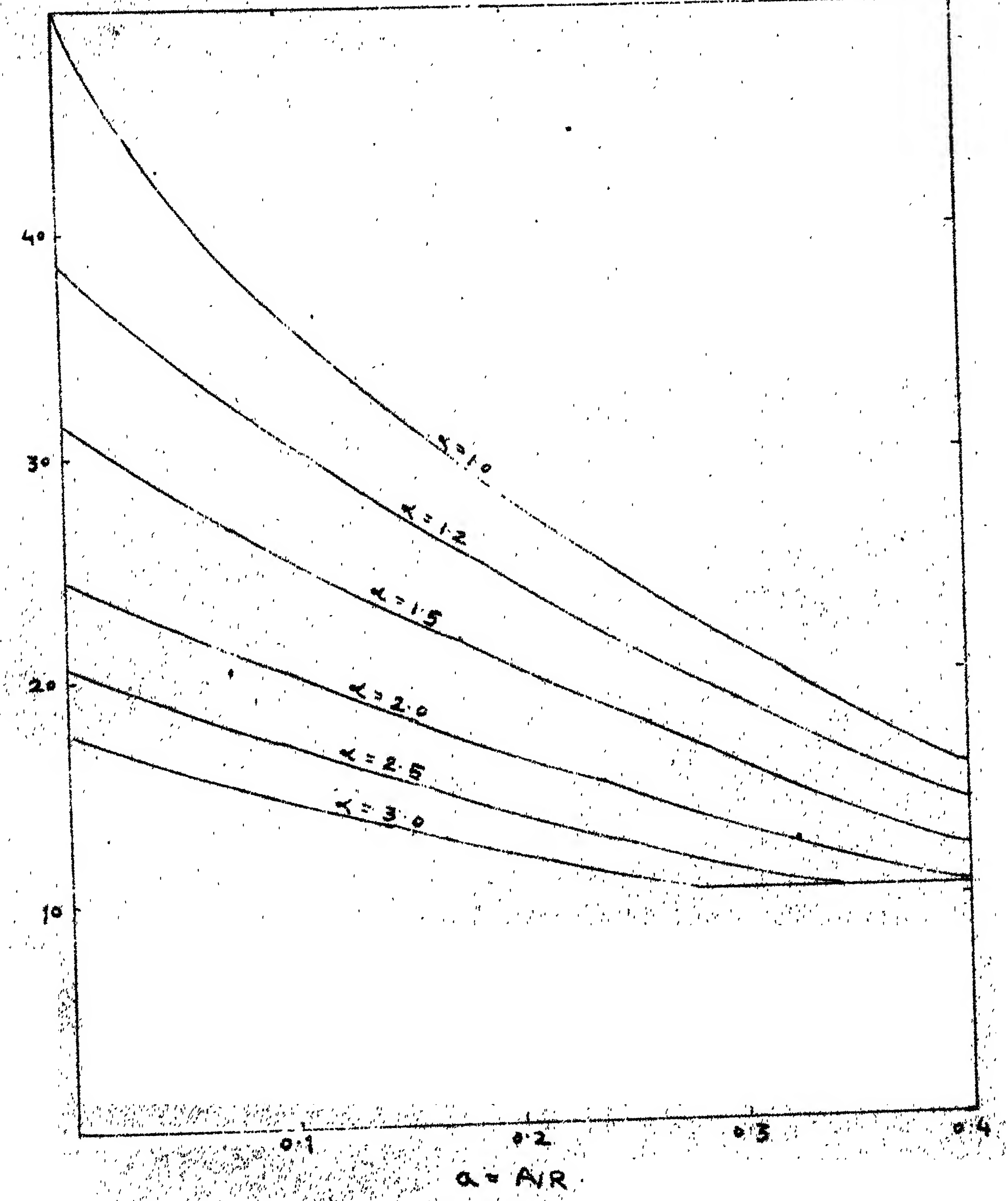
STEPPED POLAR NORMAL



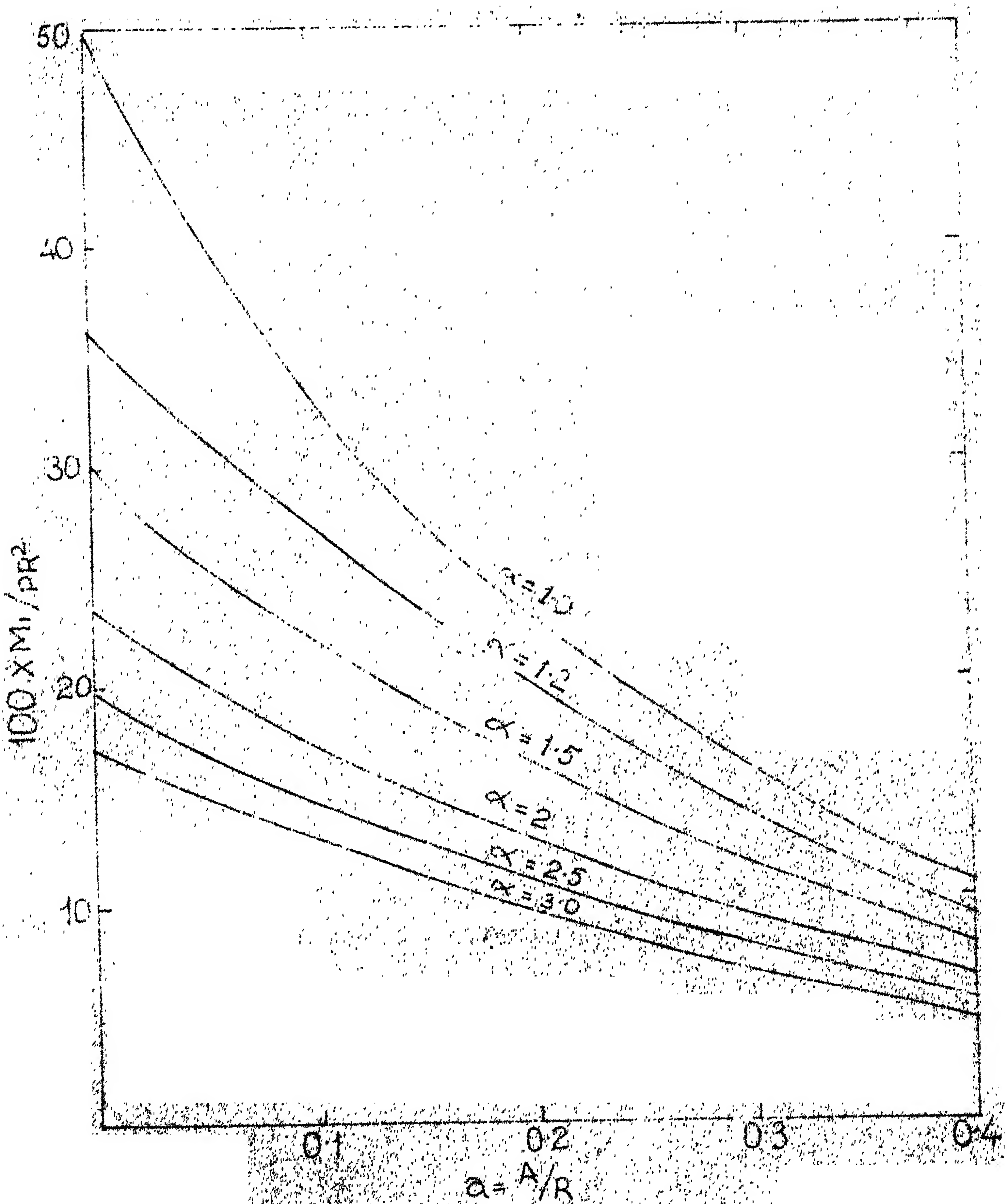
STEPPED POLAR GRANULAR



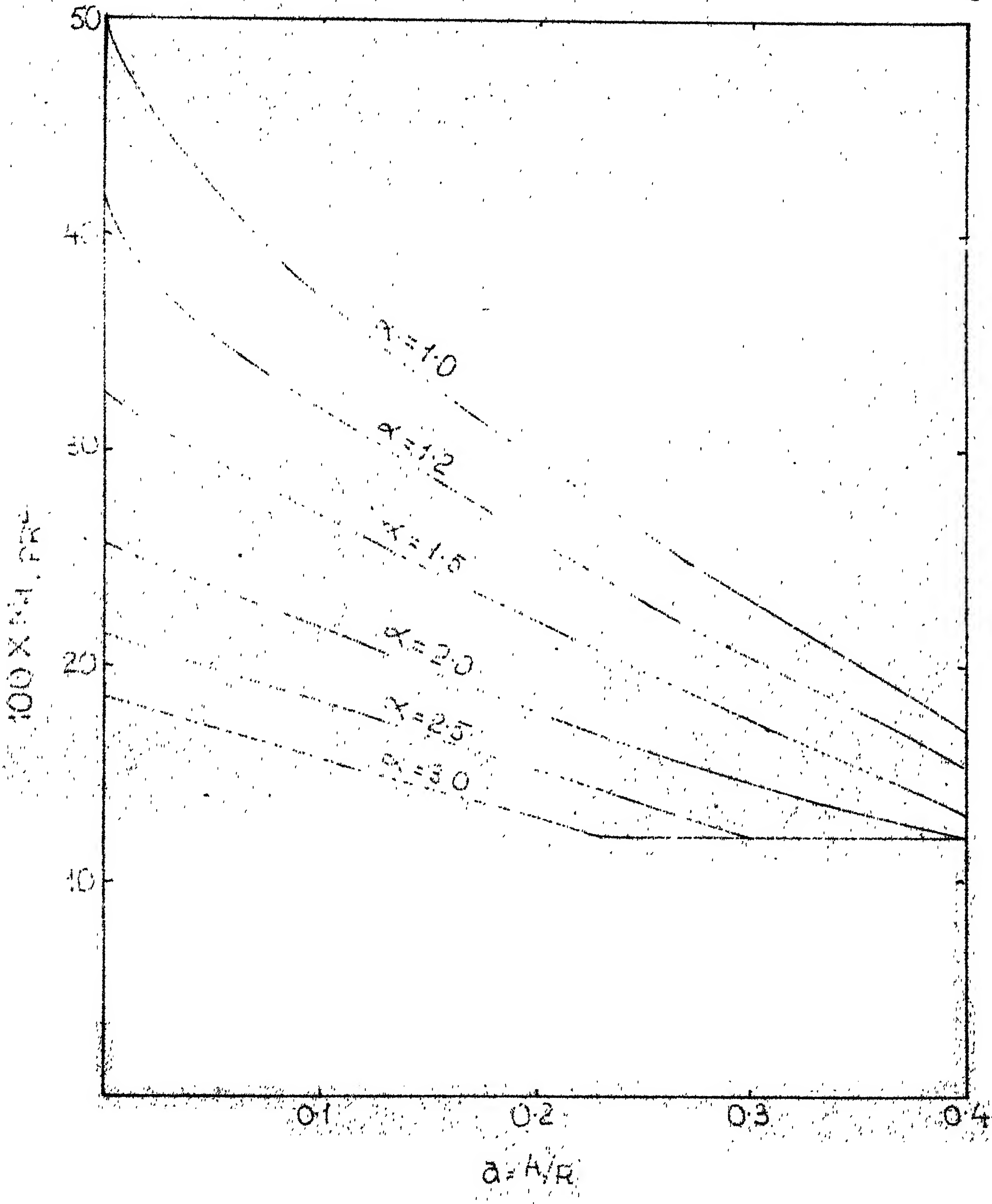
STEPPED POLAR COHESIVE



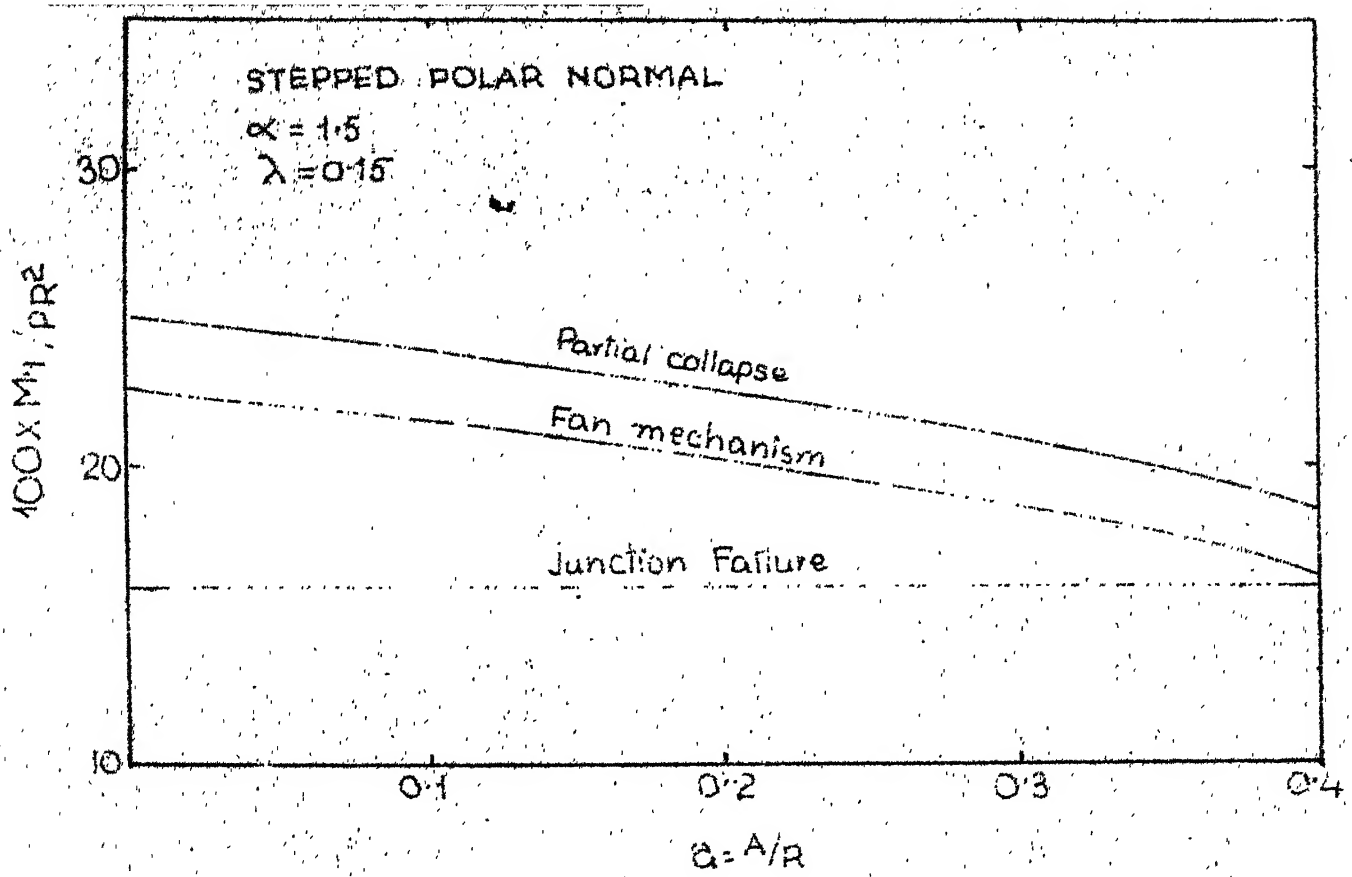
STEPPED - ISOTROPIC - NORMAL



STEPPED - ISOTROPIC - GRANULAR

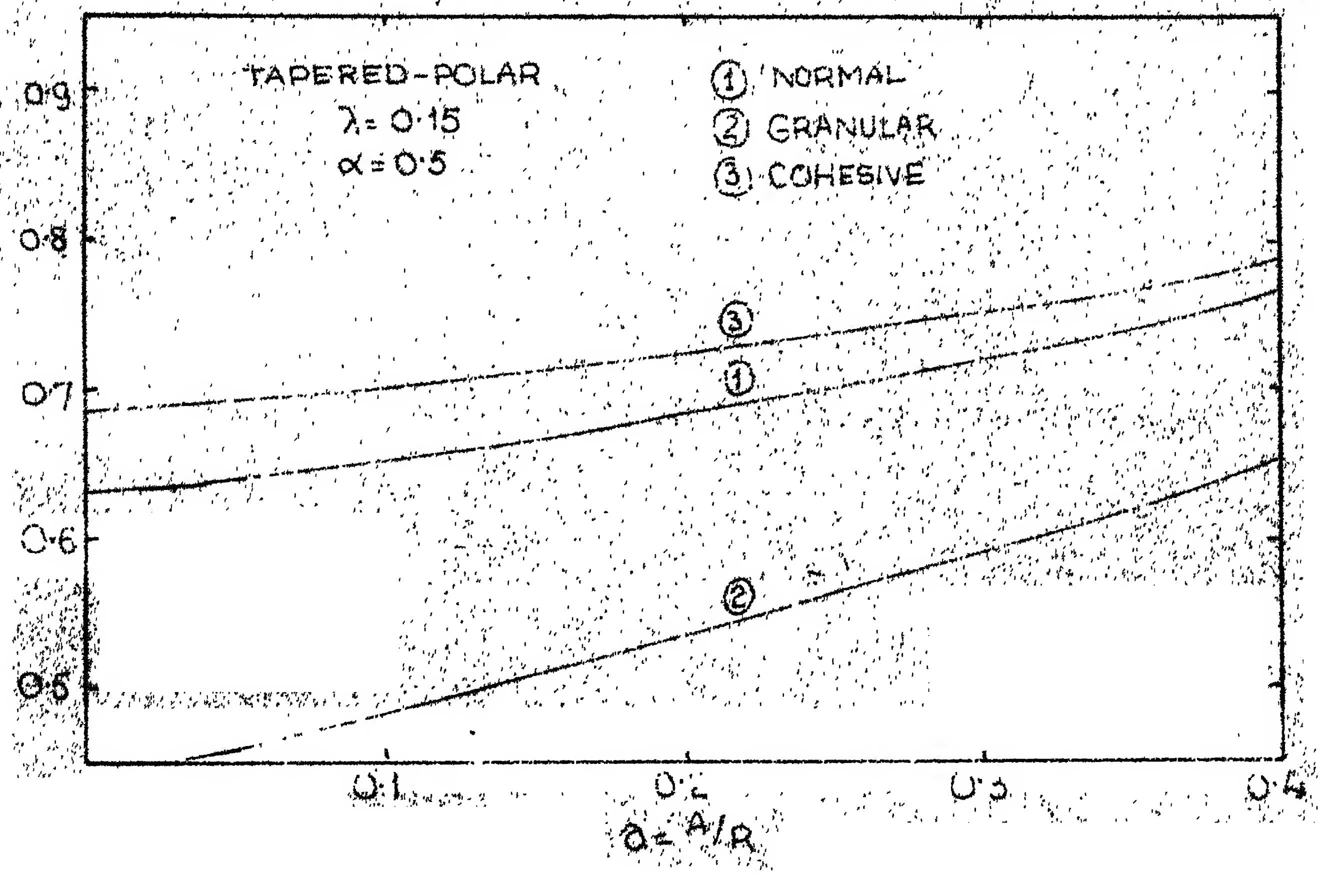


STEP ISOTROPIC COHESIVE

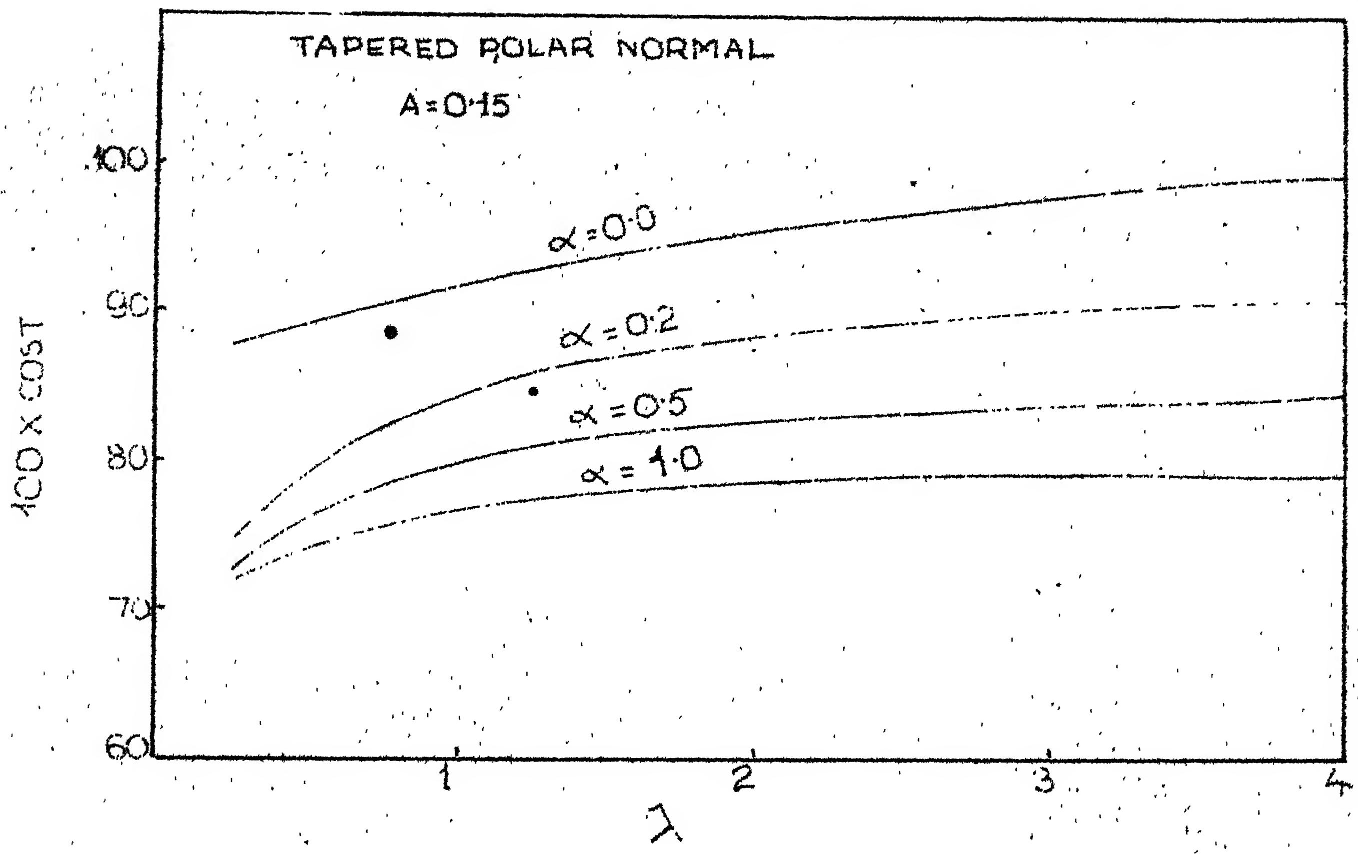


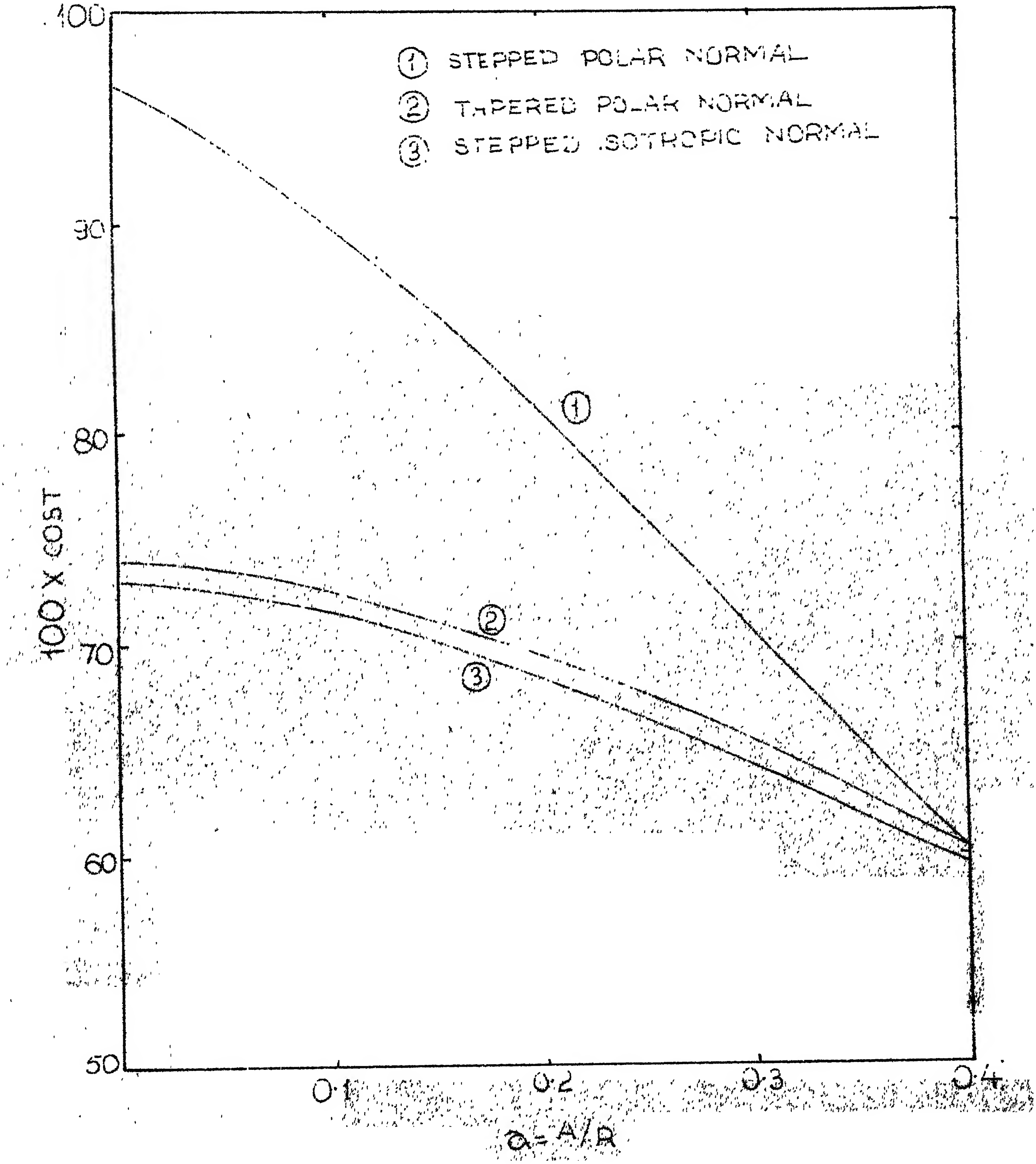
STEPPED POLAR NORMAL
α = 1·5
λ = 0·15
30
20
10
100 x M1, PR2
Partial collapse
Fan mechanism
Junction Failure
0·1
0·2
0·3
0·4
a = A/R

11



TAPERED-POLAR
λ = 0·15
α = 0·5
① NORMAL
② GRANULAR
③ COHESIVE
0·9
0·8
0·7
0·6
0·5
③
①
②
0·1
0·2
0·3
0·4
a = A/R


TAPERED POLAR NORMAL
A=0·15
α=0·0
α=0·2
α=0·5
α=1·0
100 x COST
100
90
80
70
60
1
2
3
4
λ



① STEPPED POLAR NORMAL
② TAPERED POLAR NORMAL
③ STEPPED ISOTROPIC NORMAL
100 X COST
100
90
80
70
60
50
0·1
0·2
0·3
0·4
a = A/R
①
②
③

CHAPTER 4

CONCLUSIONS

4.1 DISCUSSION OF RESULTS AND COMMENTS

Following observations have been made by inspecting Graphs 1 to 13.

1) In case of Tappered-Polar Footings the moment capacity to be provided at the edge decreases sharply as $a = A/R$ increases for lower values of α, but decreases very little for higher values of α. (Graphs 1 to 3)

2) Stepped Polar Footings follow the same trend as described in 1 for Tapered Footing and finally beyond certain value of α, moment capacity to be provided at edge remains constant as it is governed by Junction Fan Mechanism. (Graphs 4-6)

3) In case of Stepped-Isotropic Footing the moment capacity to be provided decreases sharply as $a = A/R$ increases; irrespective of the value of α but Junction Fan Mechanism finally provided a lower bound on the value of M_0/PR^2. (Graphs 7-9)

4) In case of polar net reinforcement, cost of the footing increases as λ increases. This increase is very sharp for λ upto 1. This means for economy only minimum radial reinforcement should be provided. (Graph 12)

5) As regards to economy both Tapered Polar and Stepped isotropic footings are very cheap with respect to Stepped-Polar Footing. (Graph 13)

6) Radius of the circular yield line viz. B is lowest for granular soil & highest for cohesive soil ; other conditions being same . (Graph 11)

COMMENTS

Thus we observe that Design Charts presented here can directly be used in practice with better understanding about the behaviour of the footing.

4.2 SCOPE FOR FURTHER WORK

Optimisation techniques can be used to arrive at optimum taper angles, optimum radius of the step using minimum cost criterion. Yield Equality Method can be further applied for analysing annular footings also.

APPENDIX

COST CALCULATIONS OF DIFFERENT TYPES OF FOOTINGS

Let

d_o = Depth of footing at the edge.

A_{st} = Area of cross section of steel provided per unit width.

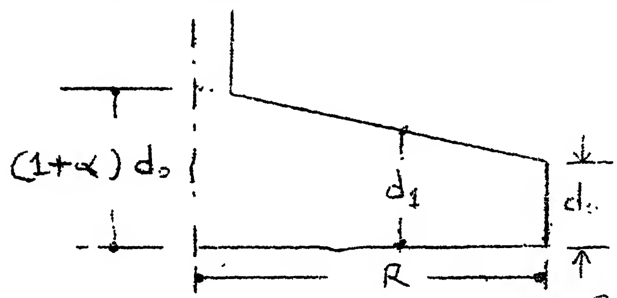
σ_{su} = Ultimate yield stress in steel

K = Lever arm factor at ultimate load

$$CR = \frac{\text{Cost of steel per unit volume}}{\text{Cost of concrete per unit volume}}$$

CD = Cost of concrete per unit volume.

A) TAPERED FOOTING WITH POLAR REINFORCEMENT



$$\text{Volume of concrete} = \int_0^R 2\pi \cdot r \cdot d_1 \cdot dr$$

$$= 2\pi d_o \int_0^R (1+\alpha(1-r/R))r \cdot dr$$

$$= 2\pi d_o \left((R^2/2)(1+\alpha) - \alpha(R^2/3)\right)$$

$$\frac{\text{Concrete volume}}{R^3} = 2\pi(d_o/R)(0.5(1+\alpha) - \alpha/3) = CV$$

Now to find volume of steel used.

If $\left(\frac{M_1}{PR^2}\right)$ is the circumferential moment capacity at the edge then,

$$A_{st} = \left(\frac{M_1}{PR^2}\right) \frac{PR^2}{\sigma_{su} \cdot d_o \cdot K}$$

at edge

Therefore

Total volume of circumferential reinforcement $= \int_A^E A_{st} \cdot 2\pi \cdot r \cdot dr$

$$= \pi R^2 \ (e^2 - a^2) \cdot \left(\frac{M_1}{PR^2}\right) \cdot \frac{PR^2}{\sigma_{su} \cdot d_o \cdot K}$$

For radial reinforcement

$$A_{st} = \frac{M_2}{PR^2} \cdot \frac{PR^2}{\sigma_{su} \cdot d_o \cdot K}$$

Total radial steel $= A_{st} \cdot 2\pi R \cdot R$

$$= \left(\frac{M_2}{PR^2}\right) \cdot \frac{PR^2 \times 2\pi R^2}{\sigma_{su} \cdot d_o \cdot K}$$

Therefore

$$\frac{\text{Total steel volume}}{R^3} = \frac{M_1}{PR^2} \cdot \frac{\pi P\,(e^2 - a^2 + 2\lambda)}{\sigma_{su} (d_o/R) \cdot K}$$

$$= ST$$

Total cost of footing $= CR \cdot CD \cdot R^3 \cdot ST + CD \cdot R^3 \cdot CV$

B) STEPPED FOOTING WITH POLAR REINFORCEMENT

$$\frac{\text{Total cost}}{CD \cdot R^3} = CR \cdot ST + CV$$

where,

$$ST = \frac{M_1}{PR^2} \cdot \frac{\pi P (c^2 - a^2 + 2\lambda)}{\sigma_{su} \cdot (d_o/R) \cdot K}$$

$$CV = \pi \cdot (d_o/R) \cdot (1 + (\alpha - 1) d^2)$$

C) STEPPED FOOTING WITH ISOTROPIC REINFORCEMENT

$$\frac{\text{Total cost}}{CD \cdot R^3} = CR \cdot ST + CV$$

where,

$$ST = \frac{(\frac{M_o}{PR^2}) (e^2 a a^2) \cdot 2\pi \cdot P}{\sigma_{su} \cdot (d_o/R) \cdot K}$$

$$CV = \pi \cdot (d_o/R) \cdot (1 + (\alpha - 1) d^2)$$

NOTE: - For the purpose of comparison of the cost of different footings following quantities are assumed as follows.

$(d_o/R) = 0.1$

$CR = 30.$

$_{su} = 26000.0 \quad T/m^2$

$P = 30.0 \quad T/m^2$

$K = 0.785$

Assuming these values cost of different footings has been calculated and graphs 12 & 13 are plotted.

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